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AND

ACCOUNTANT'S ASSISTANT.

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BY HOYT D. ORTON,
705 JAYNE STREET, PHILADELPHIA.

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INTRODUCTION.

QUANTITY is that which can be increased or diminished by augments or abatements of homogeneous parts. Quantities are of two essential kinds, *Geometrical* and *Physical.*

1. *Geometrical* quantities are those which occupy space; as *lines*, *surfaces*, *solids*, *liquids*, *gases*, etc.

2. *Physical* quantities are those which exist in the time, but occupy no space; they are known by their character and action upon geometrical quantities, as *attraction*, *light*, *heat*, *electricity* and *magnetism*, *colors*, *force*, *power*, etc.

To obtain the magnitude of a quantity we compare it with a part of the same; this part is imprinted in our mind as a *unit*, by which the whole is measured and conceived. No quantity can be measured by a quantity of another kind, but any quantity can be compared with any other quantity, and by such comparison arises what we call *calculation* or *Mathematics*

MATHEMATICS.

MATHEMATICS is a science by which the comparative value of quantities are investigated; it is divided into:

1. ARITHMETIC, that branch of Mathematics which treats of the nature and property of numbers; it is subdivided into *Addition*, *Subtraction*, *Multiplication*, *Division*, *Involution*, *Evolution and Logarithms.*

2. ALGEBRA, that branch of Mathematics which employs letters to represent quantities, and by that means performs solutions without knowing or noticing the *value* of the quantities. The subdivisions of Algebra are the same as in Arithmetic.

3. GEOMETRY, that branch of Mathematics which investigates the relative property of quantities that occupies space; its subdivisions are *Longemetry*, *Planemetry*, *Stereometry*, *Trigonometry and Conic Sections.*

4. DIFFERENTIAL-CALCULS, that branch of Mathematics which ascertains the mean effect produced by group of continued variable causes.

5. INTEGRAL-CALCULS, the contrary of Differential, or that branch of Mathematics which investigates the nature of a continued variable cause that has produced a known effect.

PREFACE.

MATHEMATICAL LAWS are the acknowledged basis of all science. Ever since the streets of Athens resounded with that historical cry of "Eureka," emanating from one of antiquity's greatest mathematicians, the science has been steadily progressing.

It is not our purpose, in this small work, to introduce any of the higher branches of mathematics, viz.: Algebra, Conic Sections, Calculus, etc. Our object is merely to present to the public a system of calculation that is practical to every business man. It consists of the addition of numbers on a principle entirely different from the one ordinarily used. In the practical application of this new principle of addition, scarcely any mental labor is required, compared with the principle of addition set forth in standard works. The superiority we claim for this principle above all others, is this, that it requires no great mental exertion, affording the

greatest facilities to the calculator in the addition of numbers, enabling him to add a whole day without any mental fatigue; whereas, by the ordinary way, it is very laborious and fatiguing.

Our system of calculation also embraces a concise rapid, and at the same time practical method of Multiplication, by which one is enabled to arrive at the product of any number of figures, multiplied by any number, immediately, without the use of partial products.

This small work also embraces the shortest and most concise method for the computation of Interest ever introduced to the public. Our system for computing interest is entirely different from any rule ever introduced, for the computation of either Simple or Compound Interest. A student having gone no further than Long Division in Arithmetic, can, by our rule, calculate Simple or Compound Interest at any given rate per cent., for any given time, in one-tenth of the time that the best calculators will compute it by the rules laid down in other books. By using our rules, you can entirely avoid the use of fractions, and save the calculation of 75 to 100 figures, where years, months and days are given on a note.

ADDITION.

To be able to add two, three or four columns of figures at once, is deemed by many to be a Herculean task, and only to be accomplished by the gifted few, or, in other words, by mathematical prodigies. If we can succeed in dispelling this illusion, it will more than repay us; and we feel very confident that we can, if the student will lay aside all prejudice, bearing steadily in mind that to become proficient in any new branch or principle a little wholesome application is necessary. On the contrary, we can not teach a student who takes no interest in the matter, one who will always be a drone in society. Such men have no need of this principle.

If two, three, or more, columns can be carried up at a time, there must be some law or rule by which it is done. We have two principles of Addition; one for adding short columns, and one for adding very long columns. They are much alike, differing only in detail. When one is thoroughly learned, it is very easy to learn the second. By a little attention to the following example, much time in future will be saved.

ADDITION OF SHORT COLUMNS OF FIGURES.

ADDITION is the basis of all numerical operations, and is used in all departments of business. To aid the business man in acquiring facility and accuracy in adding short columns of figures, the following method is presented as the best:

```
 274
 346
 134
 342
 727
 329
----
2152
```

PROCESS.—Commence at the bottom of the right-hand column, add thus: 16, 22, 32; then carry the 3 tens to the second column; then add thus: 7, 14, 25; carry the 2 hundreds to the third column, and add the same way: 12, 16, 21. In this way you name the sum of two figures at once, which is quite as easy as it is to add one figure at a time. Never permit yourself *for once* to add up a column in this manner: 9 and 7 are 16, and 2 are 18 and 4 are 22, and 6 are 28, and 4 are 32. It is just as easy to name the result of two figures at once and four times as rapid.

The following method is recommended for the

ADDITION OF LONG COLUMNS OF FIGURES.

In the addition of long columns of figures which frequently occur in books of accounts, in order to add them with certainty, and, at the same time, with ease and expedition, study well the following method, which practice will render familiar, easy, rapid, and certain.

THE EASY WAY TO ADD.

EXAMPLE 2—EXPLANATION.

Commence at 9 to add, and add as near 20 as possible, thus: 9+2+4+3=18, place the 8 to the right of the 3, as in example; commence at 6 to add 6+4+8=18; place the 8 to the right of the 8, as in example; commence at 6 to add 6+4+7=17; place the 7 to the right of the 7, as in example; commence at 4 to add 4+9+3=16; place the 6 to the right of the 3, as in example; commence at 6 to add 6+4+7=17; place the 7 to the right of the 7, as in example; now, having arrived at the top of the column, we add the figures in the new column, thus: 7+6+7+8+8=36; place the right hand figure of 36, which is a 6, under the original column, as in example, and add the left hand figure, which is a 3, to the number of figures in the new column; there are 5 figures in the new column, therefore 3+5=8; prefix the 8 with the 6, under the original column, as in example; this makes 86, which is the sum of the column.

7^7
4
6
3^6
9
4
7^7
4
6
8^5
4
6
3^8
4
2
9
—
86

Remark 1.—If, upon arriving at the top of the column, there should be one, two or three figures whose sum will not equal 10, add them on to the sum of the figures of the new column, never placing

an extra figure in the new column, unless it be an excess of units over ten.

Remark 2.—By this system of addition you can stop any place in the column, where the sum of the figures will equal 10 or the excess of 10; but the addition will be more rapid by your adding as near 20 as possible, because you will save the forming of extra figures in your new column.

EXAMPLE—EXPLANATION.

2+6+7=15, drop 10, place the 5 to the right of the 7; 6+5+4=15, drop 10, place the 5 to the right of the 4, as in example; 8+3+7=18, drop 10, place the 8 to the right of the 7, as in example; now we have an extra figure, which is 4; add this 4 to the top figure of the new column, and this sum on the balance of the figures in the new column, thus: 4+8+5+5=22; place the right hand figure of 22 under the original column, as in example, and add the left hand figure of 22 to the number of figures in the new column, which are three, thus: 2+3=5; prefix this 5 to the figure 2, under the original column; this makes 52, which is the sum of the column.

$$
\begin{array}{r}
4 \\
7^{8} \\
3 \\
8 \\
4^{5} \\
5 \\
6 \\
7^{5} \\
6 \\
2 \\
\hline
52
\end{array}
$$

RULE.—*For adding two or more columns, commence at the right hand, or units' column; proceed in the same manner as in adding one column; after the sum of the first column is obtained, add all except the right hand figure of this sum to the second column, adding the second column the same way you added the first; proceed in like manner with all the columns, always adding to each successive column the sum of the column in the next lower order, minus the right hand figure.*

N. B. The small figures which we place to the right of the column when adding are called *integers.*

The addition by integers or by forming a new column, as explained in the preceding examples should be used only in adding very long columns of figures, say a long ledger column, where the footings of each column would be two or three hundred, in which case it is superior and much more easy than any other mode of addition; but in adding short columns it would be useless to form an extra column, where there is only, say, six or eight figures to be added. In making short additions, the following suggestions will, we trust, be of use to the accountant who seeks for information on this subject.

In the addition of several columns of figures, where they are only four or five deep, or when their respective sums will range from twenty-five

to forty, the accountant should commence with the unit column, adding the sum of the first two figures to the sum of the next two, and so on, naming only the results that is the sum of every two figures.

In the present example in adding the unit 346
column instead of saying 8 and 4 are 12 and 235
5 are 17 and 6 are 23, it is better to let the 724
eye glide up the column reading only, 8, 12, 598
17, 23; and still better, instead of making a separate addition for each figure, group the figures thus: 12 and 11 are 23, and proceed in like manner with each column. For short columns this is a very expeditious way, and indeed to be preferred; but for long columns, the addition by integers is the most useful, as the mind is relieved at intervals and the mental labor of retaining the whole amount, as you add, is avoided, which is very important to any person whose mind is constantly employed in various commercial calculations.

In adding a long column, where the figures are of a medium size, that is, as many 8s and 9s as there are 2s and 3s, it is better to add about three figures at a time, because the eye will distinctly see that many at once, and the ingenious student will in a short time, if he adds by integers, be able to read the amount of three figures at a glance or as quick, we might say, as he would read a single figure.

Here we begin to add at the bottom of the unit column and add successively three figures at a time, and place their respective sums, minus 10, to the right of the last figure added; if the three figures do not make 10, add on more figures; if the three figures make 20 or more, only add two of the figures. The little figures that are placed to the right and left of the column are called integers. The integers in the present example, belonging to the units column, are 4, 4, 5, 4, 6, which we add together, making 23; place down 3 and add 2 to the number of integers, which gives 7, which we add to the tens and proceed as before.

$$
\begin{array}{r}
{}^{5}26^{6} \\
67\phantom{^{6}} \\
43\phantom{^{6}} \\
38^{4} \\
{}^{9}54\phantom{^{6}} \\
62\phantom{^{6}} \\
87^{5} \\
{}^{5}65\phantom{^{6}} \\
53\phantom{^{6}} \\
44^{4} \\
{}^{8}77\phantom{^{6}} \\
33\phantom{^{6}} \\
84^{4} \\
{}^{3}56\phantom{^{6}} \\
14\phantom{^{6}} \\
\hline
803\phantom{^{6}}
\end{array}
$$

Reason.—In the above example, every time we placed down an integer we discarded a ten, and when we set down the 3 in the answer we discarded two tens; hence, we add 2 on to the number of integers to ascertain how many tens were discarded; there being 5 integers it made 7 tens, which we now add to the column of tens; on the same principle we might add between 20 and 30, always setting down a figure before we got to 30; then every integer set down would count for 2 tens, being discarded in the same way, it does in the present instance for one ten. When we add between 10 and 20, and in very long columns, it

would be much better to go as near 30 as possible, and count 2 tens for every integer set down, in which case we would set down about one-half as many integers as when we write an integer for every ten we discard.

When adding long columns in a ledger or day-book, and where the accountant wishes to avoid the writing of extra figures in the book, he can place a strip of paper alongside of the column he wishes to add, and write the integers on the paper, and in this way the column can be added as convenient almost as if the integers were written in the book.

Perhaps, too, this would be as proper a time as any other to urge the importance of another good habit; I mean *that of making plain figures.* Some persons accustom themselves to making mere scrawls, and important blunders are often the result. If letters be badly made you may judge from such as are known; but if one figure be illegible, its value can not be inferred from the others. The vexation of the man who wrote for 2 or 3 monkeys, and had 203 sent him, was of far less importance than errors and disappointments sometimes resulting from this inexcusable practice.

We will now proceed to give some methods of proof. Many persons are fond of proving the correctness of work, and pupils are often instructed to do so, for the double purpose of giving them

exercise in calculation and saving their teacher the trouble of reviewing their work.

There are special modes of proof of elementary operations, as by casting out threes or nines, or by changing the order of the operation, as in adding upward and then downward. In Addition, some prefer reviewing the work by performing the Addition downward, rather than repeating the ordinary operation. This is better, for if a mistake be inadvertently made in any calculation, and the same routine be again followed, we are very liable to fall again into the same error. If, for instance, in running up a column of Addition you should say 84 and 8 are 93, you would be liable, in going over the same again, in the same way to slide insensibly into a similar error; but by beginning at a different point this is avoided.

This fact is one of the strongest objections to the plan of cutting off the upper line and adding it to the sum of the rest, and hence some cut off the lower line by which the spell is broken. The most thoughtless can not fail to see that adding a line *to* the sum of the rest, is the same as adding it in *with* the rest.

The mode off proof by casting out the nines and threes will be fully explained in a following chapter.

A very excellent mode of avoiding error in add-

ing long columns is to set down the result of each column on some waste spot, observing to place the numbers successively a place further to the left each time, as in putting down the product figures in multiplication; and afterward add up the amount. In this way if the operator lose his count, he is not compelled to go back to units, but only to the foot of the column on which he is operating. It is also true that the brisk accountant, who thinks on what he is doing, is less liable to err than the dilatory one who allows his mind to wander. Practice too will enable a person to read amounts without naming each figure, thus instead of saying 8 and 6 are 14, and 7 are 21 and 5 are 26, it is better to let the eye glide up the column, reading only 8, 14, 21, 26, etc.; and, still further, it is
quite practicable to accustom one's self to group 87
the figures in adding, and thus proceed very rap- 23
idly. Thus in adding the units' column, instead 45
of adding a figure at a time, we see at a glance 62
that 4 and 2 are 6, and that 5 and 3 are 8, then 24
6 and 8 are 14; we may then, if expert, add —
constantly the sum of two or three figures at a time, and with practice this will be found highly advantageous in long columns of figures; or two or three columns may be added at a time, as the practiced eye will see that 24 and 62 are 86 almost as readily as that 4 and 2 are 6.

Teachers will find the following mode of matching lines for beginners very convenient, as they can inspect them at a glance:

Add	7654384
	8786286
	3408698
	2345615
	1213713
	23408696

In placing the above the lines are matched in pairs, the digits constantly making 9. In the above, the first and fourth, second and fifth are matched; and the middle is the *key line*, the result being just like it, except the units' place, which is as many less than the units in the key line as there are pairs of lines; and a similar number will occupy the extreme left. Though sometimes used as a puzzle, it is chiefly useful in teaching learners; and as the location of the key line may be changed in each successive example, if necessary, the artifice could not be detected. The number of lines is necessarily odd.

If the student will practice the addition of long colums by integers, he will in a short time become so

proficient in its application that the forming of new columns will be unnecessary; and he will only add the number of units in excess of the tens in each column. In adding up dollars and cents in a memorandum, ledger, or day-book, the beginner should add with red ink, so as to determine readily the new column from the original.

MULTIPLICATION.

MULTIPLICATION, in its most general sense, is a series of additions of the same number; therefore, in multiplication, a number is repeated a certain number of times, and the result thus obtained is called the product. When the multiplicand and the multiplier are each composed of only two figures, to ascertain the product we have the following

RULE.—*Set down the smaller factor under the larger, units under units, tens under tens. Begin with the unit figure of the multiplier, multiply by it, first the units of the multiplicand, setting the units of the product, and reserving the tens to be added to the next product; now multiply the tens of the multiplicand by the unit figure of the multiplier, and the units of the multiplicand by tens figure of*

the multiplier; add these two products together, setting down the units of their sum, and reserving the tens to be added to the next product; now multiply the tens of the multiplicand by the tens figure of the multiplier, and set down the whole amount. This will be the complete product.

Remark.—Always add in the tens that are reserved as soon as you form the first product.

EXAMPLE 1.—EXPLANATION.

1. Multiply the units of the multiplicand	24
by the unit figure of the multiplier, thus:	31
1×4 is 4; set the 4 down as in example.	——
2. Multiply the tens in the multiplicand by	744

the unit figure in the multiplier, and the units in the multiplicand by the tens figure in the multiplier, thus: 1×2 is 2; 3×4 are 12, add these two products together, $2+12$ are 14, set the 4 down as in example, and reserve the 1 to be added to the next product. 3. Multiply the tens in the multiplicand by the tens figure in the multiplier, and add in the tens that were reserved, thus: 3×2 are 6, and $6+1=7$; now set down the whole amount, which is 7.

EXAMPLE 2.—EXPLANATION.

1. Multiply units by units, thus: 4×3	53
are 12, set down the 2 and reserve the 1 to	84
carry. 2. Multiply tens by units, and units	——
by tens, and add in the one to carry on the	4452

first product, then add these two products together, thus: 4×5 are $20+1$ are 21, and 8×3 are 24, and $21+24$ are 45, set down the 5 and reserve the 4 to carry to the next product. 3. Multiply tens by tens, and add in what was reserved to carry, thus: 8×5 are $40+4$ are 44, now set down the whole amount, which is 44.

EXAMPLE 3.—EXPLANATION.

5×3 are 15, set down the 5 and carry the 1 to the next product; 5×4 are $20=1$ are 21; 2×3 are 6, $21+6$ are 27, set down the 7 and carry the 2; 2×4 are $8+2$ are 10; now set down the whole amount.

$$\begin{array}{r} 43 \\ 25 \\ \hline 1075 \end{array}$$

When the multiplicand is composed of three figures, and there are only two figures in the multiplier, we obtain the product by the following

RULE.—*Set down the smaller factor under the larger, units under units, tens under tens; now multiply the first upper figure by the unit figure of the multiplier, setting down the units of the product, and reserving the tens to be added to the next product; now multiply the second upper by units, and the first upper by tens, add these two products together, setting down the units figure of their sum, and reserving the tens to carry, as before; now multiply the third upper by units, and the second upper by tens, add these two products together, setting down the units figure of their sum, and reserving the tens to*

carry, as usual; now multiply the third upper by tens, add in the reserved figure, if there is one, and set down the whole amount. This will be the complete product.

Remark.—One of the principal errors with the beginner, in this system of multiplication, is neglecting to add in the reserved figure. The student must bear in mind that the reserved figure is added on to the first product obtained after the setting down of a figure in the complete product.

EXAMPLE 1.—EXPLANATION.

$$\begin{array}{r} 123 \\ 45 \\ \hline 5535 \end{array}$$

Multiply first upper by units, 5×3 are 15, set down the 5, reserve the 1 to carry to the next product; now multiply second upper by units and first upper by tens, 5×2 are 10+1 are 11, 4×3 are 12, add these products together; 11+12 are 23, set down the 3, reserve the 2 to carry; now multiply third upper by units, and second upper by tens, add these two products together, always adding on the reserved figure to the first product; 5×1 are 5+2 are 7, 4×2 are 8, and 7+8 are 15, set down the 5, reserve the 1; now multiply third upper by tens, and set down the whole amount; 4×1 are 4+1 are 5, set down the 5. This will give the complete product.

Multiply 32 by 45 in a single line.

Here we multiply 5×2 and set down and carry as usual; then to what you carry add 5×3 and 4×2, which gives 24; set down 4 and carry 2 to 4×3, which gives 14 and completes the product.

$$\begin{array}{r} 32 \\ 45 \\ \hline 1440 \end{array}$$

Multiply 123 by 456 in a single line.

Here the first and second places are found as before; for the third, add 6×1, 5×2, 4×3, with the 2 you had to carry, making 30; set down 0 and carry 3; then drop the units' place and multiply the hundreds and tens crosswise, as you did the tens and units, and you find the thousand figure; then, dropping both units and tens, multiply the 4×1, adding the 1 you carried, and you have 5, which completes the product. The same principle may be extended to any number of places; but let each step be made perfectly familiar before advancing to another. Begin with two places, then take three, then four, but always practicing some time on each number, for any hesitation as you progress will, confuse you.

$$\begin{array}{r} 123 \\ 456 \\ \hline 56088 \end{array}$$

N. B. The following mode of multiplying numbers will only apply where the sum of the two last or unit figures equal ten, and the other figures in both factors are the same.

CONTRACTIONS IN MULTIPLICATION.

To multiply when the unit figures added equal (10) *and the tens are alike as* 72 *by* 78, *&c.*

1st. Multiply the units and set down the result.

2d. Add 1 to either number in tens place and multiply by the other, and you have the complete product.

EXAMPLE FIRST—PROCESS.

Here because the sum of the units 4 and 6 are *ten* and the *tens* are alike; we simply say 4 times 6 are 24, and set down both figures of the product; then because 4 and 6 make *ten* we add 1 to 8, making 9, and 9 times 8 are 72, which completes the product.

$$\begin{array}{r} 86 \\ 84 \\ \hline 7224 \end{array}$$

NOTE.—If the product of units do not contain ten the place of tens must be filled with a cipher

The above rule is useful in examples like the fol-

2. What will 93 acres of land cost at 97 dollars

lowing:

per acre? Ans. $9021.

3. What will 89 pounds of tea cost at 81 cents per pound? Ans. $72.09.

In the above the product of 9 *by* 1 *did not amount to ten, therefore* 0 *is placed in tens place.*

4. Multiply 998 by 992. Ans. 990016.

In the above, because 2 *and* 8 *are* 10, *we add* 1 *to* 99, *making* 100; *then* 100 *times* 99 *are* 9900.

EXAMPLE EIGHTEENTH.

Multiply 79 by 71 in a single line.

Here we multiply 1×9 and set down the result, then we multiply the 7 in the multiplicand, increased by 1 by the 7 in the multiplier, 7×8, which gives 56 and completes the product.

$$\begin{array}{r} 79 \\ 71 \\ \hline 5609 \end{array}$$

EXAMPLE NINETEENTH.

Multiply 197 by 193 in a single line.

Here we multiply 3×7 and set down the result, then we multiply the 19 in the multiplicand, increased by 1 by the 19 in the multiplier, 19×20, which gives 380 and completes the product.

$$\begin{array}{r} 197 \\ 193 \\ \hline 38021 \end{array}$$

EXAMPLE TWENTIETH.

Multiply 996 by 994 in a single line.

Here we multiply 4×6 and set down the result, then we multiply the 99 in the multiplicand, increased by 1 by the 99 in the multiplier, 99×100, which gives 9900 and completes the product.

$$\begin{array}{r} 996 \\ 994 \\ \hline 990024 \end{array}$$

EXAMPLE TWENTY-FIRST.

Multiply 1208 by 1202 in a single line.

Here we multiply 2×8 and set down the result, then we multiply the 120 in the multiplicand, increased by 1 by the 120 in the multiplier, 120×121, which gives 14520 and completes the product.

$$\begin{array}{r} 1208 \\ 1202 \\ \hline 1452016 \end{array}$$

CURIOUS AND USEFUL CONTRACTIONS.

To multiply any number, of two figures, by 11,

RULE.— *Write the sum of the figures between them.*

1. Multiply 45 by 11. Ans. 495

Here 4 and 5 are 9, which write between 4 & 5.

2. Multiply 34 by 11. Ans. 374.

N. B. When the sum of the two figures is over 9, increase the left-hand figure by the 1 to carry.

3. Multiply 87 by 11. Ans. 957.

To square any number of 9s instantaneously, and without multiplying,

RULE.— *Write down as many 9s less one as there are 9s in the given number, an 8, as many 0s as 9s, and a 1.*

4. What is the square of 9999? Ans. 99980001.

EXPLANATION.—We have four 9s in the given number, so we write down three 9s, then an 8, then three 0s, and a 1.

5. Square 999999. Ans. 999998000001.

To square any number ending in 5,

RULE.—*Omit the 5 and multiply the number, as it will then stand by the next higher number, and annex 25 to the product.*

6. What is the square of 75? Ans. 5625.

EXPLANATION.—We simply say, 7 times 8 are 56, to which we annex 25.

7. What is the square of 95? Ans. 9025

Mental Operations in Fractions.

To square any number containing $\frac{1}{2}$, as $6\frac{1}{2}$, $9\frac{1}{2}$,

RULE.—*Multiply the whole number by the next higher whole number, and annex $\frac{1}{4}$ to the product.*

Ex. 1. What is the square of $7\frac{1}{2}$? Ans. $56\frac{1}{4}$.

We simply say, 7 times 8 are 56, to which we add $\frac{1}{4}$.

2. What will $9\frac{1}{2}$ lbs. beef cost at $9\frac{1}{2}$ cts. a lb.?
3. What will $12\frac{1}{2}$ yds. tape cost at $12\frac{1}{2}$ cts. a yd.?
4. What will $5\frac{1}{2}$ lbs. nails cost at $5\frac{1}{2}$ cts. a lb.?
5. What will $11\frac{1}{2}$ yds. tape cost at $11\frac{1}{2}$ cts. a yd.?
6. What will $19\frac{1}{2}$ bu. bran cost at $19\frac{1}{2}$ cts. a bu.?

REASON.—We multiply the whole number by the next higher whole number, because half of any number taken twice and added to its square is the same as to multiply the given number by ONE more than itself. The same principle will multiply any two *like* numbers together, when the sum of the fractions is ONE, as $8\frac{1}{3}$ by $8\frac{2}{3}$, or $11\frac{3}{8}$ by $11\frac{5}{8}$, etc It is obvious that to multiply any number by any two fractions whose sum is ONE, that the sum of the products *must be the original number*, and adding the number to its square is simply to multiply it by ONE more than itself; for instance, to multiply $7\frac{1}{4}$ by $7\frac{3}{4}$, we simply say, 7 times 8 are 56, and then, to complete the multiplication, we add, of course, the product of the fractions ($\frac{3}{4}$ times $\frac{1}{4}$ are $\frac{3}{16}$), making $56\frac{3}{16}$ the answer.

Where the sum of the Fractions is ONE.

To multiply any two *like* numbers together when the sum of the fractions is ONE.

RULE.—*Multiply the whole number by the next higher whole number; after which, add the product of the fractions.*

N. B. In the following examples, the product of the fractions are obtained *first* for convenience.

PRACTICAL EXAMPLES FOR BUSINESS MEN.

Multiply $3\frac{3}{4}$ by $3\frac{1}{4}$ in a single line.

Here we multiply $\frac{1}{4}\times\frac{3}{4}$, which gives $\frac{3}{16}$, and set down the result; then we multiply the 3 in the multiplicand, increased by unity, by the 3 in the multiplier, 3×4, which gives 12 and completes the product.

$$\begin{array}{r} 3\frac{3}{4} \\ 3\frac{1}{4} \\ \hline 12\frac{3}{16} \end{array}$$

Multiply $7\frac{2}{5}$ by $7\frac{3}{5}$ in a single line.

Here we multiply $\frac{3}{5}\times\frac{2}{5}$, which gives $\frac{6}{25}$, and set down the result; then we multiply the 7 in the multiplicand, increased by unity, by the 7 in the multiplier, 7×8, which gives 56, and completes the product.

$$\begin{array}{r} 7\frac{2}{5} \\ 7\frac{3}{5} \\ \hline 56\frac{6}{25} \end{array}$$

Multiply $11\frac{1}{3}$ by $11\frac{2}{3}$ in a single line.

Here we multiply $\frac{2}{3}\times\frac{1}{3}$, which gives $\frac{2}{9}$, and set down the result; then we multiply the 11 in the multiplicand, increased by unity, by the 11 in the multiplier, 11×12, which gives 132, and completes the product.

$$\begin{array}{r} 11\frac{1}{3} \\ 11\frac{2}{3} \\ \hline 132\frac{2}{9} \end{array}$$

EXAMPLE THIRTY-THIRD.

Multiply $16\frac{2}{3}$ by $16\frac{1}{3}$ in a single line.

Here we multiply $\frac{1}{3}\times\frac{2}{3}$ which gives $\frac{2}{9}$, and set down the result, then we multiply the 16 in the multiplicand, increased by unity by the 16 in the multiplier, 16×17, which gives 272 and completes the product.

$$\begin{array}{r} 16\frac{2}{3} \\ 16\frac{1}{3} \\ \hline 272\frac{2}{9} \end{array}$$

EXAMPLE THIRTY-FOURTH.

Multiply $29\frac{1}{2}$ by $29\frac{1}{2}$ in a single line.

Here we multiply $\frac{1}{2}\times\frac{1}{2}$ which gives $\frac{1}{4}$, and set down the result, then we multiply the 29 in the multiplicand, increased by unity by the 29 in the multiplier, 29×30, which gives 870 and completes the product.

$$\begin{array}{r} 29\frac{1}{2} \\ 29\frac{1}{2} \\ \hline 870\frac{1}{4} \end{array}$$

EXAMPLE THIRTY-FIFTH.

Multiply $999\frac{3}{8}$ by $999\frac{5}{8}$ in a single line.

Here we multiply $\frac{5}{8}\times\frac{3}{8}$, which gives $\frac{15}{64}$, and set down the result, then we multiply the 999 in the multiplicand, increased by unity by the 999 in the multiplier, 999×1000, which gives 999000 and completes the product.

$$\begin{array}{r} 999\frac{3}{8} \\ 999\frac{5}{8} \\ \hline 999000\frac{15}{64} \end{array}$$

NOTE.—The system of multiplication introduced in the preceding examples, applies to all numbers. Where the sum of the fractions is *one*, and the whole numbers are alike, or differ by *one*, the learner is requested to study well these useful properties of numbers.

Where the sum of the Fractions is ONE.

To multiply any two numbers whose difference is *one*, and the sum of the fractions is *one*,

RULE.—*Multiply the larger number, increased by* ONE, *by the smaller number; then square the fraction of the larger number, and subtract its square from* ONE.

PRACTICAL EXAMPLES FOR BUSINESS MEN.

1. What will $9\frac{1}{4}$ lbs. sugar cost at $8\frac{3}{4}$ cts. a lb.?

Here we multiply 9, increased by 1, by 8, thus, 8×10 are 80, and set down the result; then from 1 we subtract the square of $\frac{1}{4}$, thus, $\frac{1}{4}$ squared is $\frac{1}{16}$, and 1 less $\frac{1}{16}$ is $\frac{15}{16}$.

$$\begin{array}{r} 9\frac{1}{4} \\ 8\frac{3}{4} \\ \hline 80\frac{15}{16} \end{array}$$

2. What will $8\frac{2}{3}$ bu. coal cost at $7\frac{1}{3}$ cts. a bu.?

Here we multiply 8, increased by 1, by 7, thus, 7 times 9 are 63, and set down the result; then from 1 we subtract the square of $\frac{2}{3}$, thus, $\frac{2}{3}$ squared is $\frac{4}{9}$, and 1, less $\frac{4}{9}$, is $\frac{5}{9}$.

$$\begin{array}{r} 8\frac{2}{3} \\ 7\frac{1}{3} \\ \hline 63\frac{5}{9} \end{array}$$

3. What will $11\frac{2}{13}$ bu. seed cost at $\$10\frac{11}{13}$ a bu.?

Here we multiply 11, increased by 1, by 10, thus, 10 times 12 are 120, and set down the result; then from 1 we subtract the square of $\frac{2}{13}$, thus, $\frac{2}{13}$ squared is $\frac{4}{169}$, and 1 less $\frac{4}{169}$ is $\frac{165}{169}$.

$$\begin{array}{r} 11\frac{2}{13} \\ 10\frac{11}{13} \\ \hline 120\frac{165}{169} \end{array}$$

4. How many square inches in a floor $99\frac{3}{8}$ in. wide and $98\frac{5}{8}$ in. long? Ans. $9800\frac{55}{64}$.

METHOD OF OPERATION.

EXAMPLE FIRST.

Multiply $6\frac{1}{4}$ by $6\frac{1}{4}$ in a single line.

Here we add $6\frac{1}{4}+\frac{1}{4}$, which gives $6\frac{1}{2}$; this multiplied by the 6 in the multiplier, $6\times6\frac{1}{2}$, gives 39, to which we add the product of the fractions, thus $\frac{1}{4}\times\frac{1}{4}$ gives $\frac{1}{16}$, added to 39 completes the product.

$$\begin{array}{r} 6\frac{1}{4} \\ 6\frac{1}{4} \\ \hline 39\frac{1}{16} \end{array}$$

EXAMPLE SECOND.

Multiply $11\frac{1}{4}$ by $11\frac{3}{4}$ in a single line.

Here we would add $11\frac{1}{4}+\frac{3}{4}$, which gives 12; this multiplied by the 11 in the multiplier gives 132, to which we add the product of the fractions, thus $\frac{3}{4}\times\frac{1}{4}$ gives $\frac{3}{16}$, which added to 132 completes the product.

$$\begin{array}{r} 11\frac{1}{4} \\ 11\frac{3}{4} \\ \hline 132\frac{3}{16} \end{array}$$

EXAMPLE THIRD.

Multiply $12\frac{1}{2}$ by $12\frac{3}{4}$ in a single line.

Here we add $12\frac{1}{2}+\frac{3}{4}$, which gives $13\frac{1}{4}$; this multiplied by the 12 in the multiplier, $12\times13\frac{1}{4}$, gives 159, to which add the product of the fractions, thus $\frac{3}{4}\times\frac{1}{2}$ gives $\frac{3}{8}$, which added to 159 completes the product.

$$\begin{array}{r} 12\frac{1}{2} \\ 12\frac{3}{4} \\ \hline 159\frac{3}{8} \end{array}$$

Where the Fractions have a Like Denominator.

To multiply any two *like* numbers together, each of which has a fraction with a *like* denominator, as $4\frac{3}{8}$ by $4\frac{7}{8}$, or $11\frac{1}{4}$ by $11\frac{3}{4}$, or $10\frac{2}{5}$ by $10\frac{1}{5}$, etc.

Rule.—*Add to the multiplicand the fraction of the multiplier, and multiply this sum by the whole number; after which, add the product of the fractions*

PRACTICAL EXAMPLES FOR BUSINESS MEN.

N. B. In the following example, the sum of the fractions is one.

1. What will $9\frac{3}{4}$ lbs. beef cost at $9\frac{1}{4}$ cts. a lb.?

The sum of $9\frac{3}{4}$ and $\frac{1}{4}$ is ten, so we simply say, 9 times 10 are 90; then we add the product of the fractions, $\frac{1}{4}$ times $\frac{3}{4}$ are $\frac{3}{16}$.

$$\begin{array}{r} 9\frac{3}{4} \\ 9\frac{1}{4} \\ \hline 90\frac{3}{16} \end{array}$$

N. B. In the following example, the sum of the fractions is *less* than one.

2 What will $8\frac{1}{4}$ yds. tape cost at $8\frac{2}{4}$ cts. a yd.?

The sum of $8\frac{1}{4}$ and $\frac{2}{4}$ is $8\frac{3}{4}$, so we simply say, 8 times $8\frac{3}{4}$ are 70; then we add the product of the fractions, $\frac{2}{4}$ times $\frac{1}{4}$ are $\frac{2}{16}$ or $\frac{1}{8}$.

$$\begin{array}{r} 8\frac{1}{4} \\ 8\frac{2}{4} \\ \hline 70\frac{1}{8} \end{array}$$

N. B. In the following example, the sum of the fractions is *greater* than one.

3. What will $4\frac{3}{8}$ yds. cloth cost at $\$4\frac{7}{8}$ a yd.?

The sum of $4\frac{3}{8}$ and $\frac{7}{8}$ is $5\frac{1}{4}$, so we simply say, 4 times $5\frac{1}{4}$ are 21; then we add the product of the fractions, $\frac{7}{8}$ times $\frac{3}{8}$ are $\frac{21}{64}$.

$$\begin{array}{r} 4\frac{3}{8} \\ 4\frac{7}{8} \\ \hline 21\frac{21}{64} \end{array}$$

N. B. Where the fractions have different denominators, reduce them to a common denominator.

Rapid Process of Multiplying Mixed Numbers.

A valuable and useful rule for the accountant in the practical calculations of the counting-room.

To multiply any two numbers together, each of which involves the fraction $\frac{1}{2}$, as $7\frac{1}{2}$ by $9\frac{1}{2}$, etc.,

RULE.—*To the product of the whole numbers add half their sum plus* $\frac{1}{4}$.

EXAMPLES FOR MENTAL OPERATIONS.

1. What will $3\frac{1}{2}$ doz. eggs cost at $7\frac{1}{2}$ cts. a doz.?

Here the sum of 7 and 3 is 10, and half this sum is 5, so we simply say, 7 times 3 are 21 and 5 are 26, to which we add $\frac{1}{4}$.

$$\begin{array}{r} 3\frac{1}{2} \\ 7\frac{1}{2} \\ \hline 26\frac{1}{4} \end{array}$$

N. B. If the sum be an odd number, call it one less to make it even, and in such cases the fraction must be $\frac{3}{4}$.

2. What will $11\frac{1}{2}$ lbs. cheese cost at $9\frac{1}{2}$ cts. a lb.?
3. What will $8\frac{1}{2}$ yds. tape cost at $15\frac{1}{2}$ cts. a yd.?
4. What will $7\frac{1}{2}$ lbs. rice cost at $13\frac{1}{2}$ cts. a lb.?
5. What will $10\frac{1}{2}$ bu. coal cost at $12\frac{1}{2}$ cts. a bu.?

REASON.—In explaining the above rule, we add half their sum because half of either number added to half the other would be half their sum, and we add $\frac{1}{4}$ because $\frac{1}{2}$ by $\frac{1}{2}$ is $\frac{1}{4}$. The same principle will multiply any two numbers together, each of which has the same fraction; for instance, if the fraction was $\frac{1}{3}$, we would add one-third their sum; if $\frac{3}{4}$, we would add three-fourths their sum, etc.; and then, to complete the multiplication, we would add, of course, the product of the fractions.

GENERAL RULE

For multiplying any two numbers together, each of which involves the same fraction.

To the product of the whole numbers, add the product of their sum by either fraction; after which add the product of their fractions.

EXAMPLES FOR MENTAL OPERATIONS.

1. What will $11\frac{3}{4}$ lbs. rice cost at $9\frac{3}{4}$ cts. a lb.?

Here the sum of 9 and 11 is 20, and three-fourths of this sum is 15, so we simply say, 9 times 11 are 99 and 15 are 114, to which we add the product of the fractions $(\frac{9}{16})$.

$$\begin{array}{r} 11\frac{3}{4} \\ 9\frac{3}{4} \\ \hline 114\frac{9}{16} \end{array}$$

2. What will $7\frac{2}{3}$ doz. eggs cost at $8\frac{2}{3}$ cts. a doz.?
3. What will $6\frac{3}{4}$ bu. coal cost at $6\frac{3}{4}$ cts. a bu.?
4. What will $45\frac{3}{4}$ bu. seed cost at $3\frac{3}{4}$ dol. a bu.?
5. What will $3\frac{3}{8}$ yds. cloth cost at $5\frac{3}{8}$ dol. a yd.?
6. What will $17\frac{2}{5}$ ft. boards cost at $13\frac{2}{5}$ cts a ft.?
7. What will $18\frac{3}{4}$ lbs. butter cost at $18\frac{3}{4}$ cts. a lb.?

N. B. If the product of the sum by either fraction is a whole number with a fraction, it is better to reserve the fraction until we are through with the whole numbers, and then add it to the product of the fractions; for instance, to multiply $3\frac{1}{4}$ by $7\frac{1}{4}$, we find the sum of 7 and 3, which is 10, and one fourth of this sum is $2\frac{1}{2}$; setting the $\frac{1}{2}$ down in some waste spot, we simply say, 7 times 3 are 21 and 2 are 23; then, adding the $\frac{1}{2}$ to the product of the fractions $(\frac{1}{16})$, gives $\frac{9}{16}$, making $23\frac{9}{16}$, Ans

Rapid Process of Multiplying all Mixed Numbers.

N. B. Let the student remember that this is a general and universal rule.

GENELAL RULE.

To multiply any two mixed numbers together,

1st. *Multiply the whole numbers together.*

2d. *Multiply the upper digit by the lower fraction.*

3d. *Multiply the lower digit by the upper fraction.*

4th. *Multiply the fractions together.*

5th. *Add these* FOUR *products together.*

N. B. This rule is so simple, so useful, and so true that every banker, broker, merchant, and clerk should post it up for reference and use.

PRACTICAL EXAMPLES FOR BUSINESS MEN

N. B. The following method is recommended to beginners:

EXAMPLE.—Multiply $12\frac{2}{3}$ by $9\frac{3}{4}$.	$12\frac{2}{3}$
1st. We multiply the whole numbers.	$9\frac{3}{4}$
2d. Multiply 12 by $\frac{3}{4}$ and write it down.	108
3d. Multiply 9 by $\frac{2}{3}$ and write it down.	9
4th. Multiply $\frac{3}{4}$ by $\frac{2}{3}$ and write it down.	6
5th. Add these *four* products together,	$0\frac{6}{12}$
and we have the complete result.	$123\frac{6}{12}$

N. B. When the student has become familiar with the above process, it is better to do the intermediate work in the head, and, instead of setting down the partial products, add them in the mind as you pass along, and thus proceed very rapidly.

Multiply $8\frac{1}{5}$ by $10\frac{1}{4}$.

Here we simply say 10 times 8 are 80 and $\frac{1}{4}$ of 8 is 2, making 82, and $\frac{1}{5}$ of 10 is 2, which makes 84; then $\frac{1}{4}$ times $\frac{1}{5}$ is $\frac{1}{20}$, making $84\frac{1}{20}$ the answer.

$$\begin{array}{r} 8\frac{1}{5} \\ 10\frac{1}{4} \\ \hline 84\frac{1}{20} \end{array}$$

PRACTICAL BUSINESS METHOD

For Multiplying all Mixed Numbers.

Merchants, grocers, and business men generally, in multiplying the mixed numbers that arise in the practical calculations of their business, only care about having the answer correct to the nearest cent; that is, they disregard the fraction. When it is a half cent or more, they call it another cent; if less than half a cent, they drop it. And the object of the following rule is to show the business man the easiest and most rapid process of finding the product to the nearest unit of any two numbers, one or both of which involves a fraction.

GENERAL RULE.

To multiply any two numbers to the nearest unit,

1st. *Multiply the whole number in the multiplicand by the fraction in the multiplier to the nearest unit.*

2d. *Multiply the whole number in the multiplier by the fraction in the multiplicand to the nearest unit*

3d. *Multiply the whole numbers together and add the three products in your mind as you proceed.*

N. B. In actual business the work can generally be done mentally for only easy fractions *occur in business.*

N B. This rule is so simple and so true, according to all business usage, that every accountant should make himself perfectly familiar with its application. There being no such thing as a fractian to add in, there is scarcely any liability to error or mistake. By no other arithmetical process can the result be obtained by so few figures.

EXAMPLES FOR MENTAL OPERATION.

EXAMPLE FIRST.

Multiply $11\frac{1}{3}$ by $8\frac{1}{4}$ by business method.

Here $\frac{1}{4}$ of 11 to the nearest unit is 3, and $\frac{1}{3}$ of 8 to the nearest unit is 3, making 6, so we simply say, 8 times 11 are 88 and 6 are 94, Ans.

$$\begin{array}{r} 11\frac{1}{3} \\ 8\frac{1}{4} \\ \hline 94 \end{array}$$

REASON.—$\frac{1}{4}$ of 11 is nearer 3 than 2, and $\frac{1}{3}$ of 8 is nearer 3 than 2. Make the nearest whole number the quotient.

EXAMPLE SECOND.

Multiply $7\frac{3}{4}$ by $9\frac{2}{5}$ by business method.

Here $\frac{2}{5}$ of 7 to the nearest unit is 3, and $\frac{3}{4}$ of 9 to the nearest unit is 7; then 3 plus 7 is 10, so we simply say, 9 times 7 are 63 and 10 are 73, Ans.

$$\begin{array}{r} 7\frac{3}{4} \\ 9\frac{2}{5} \\ \hline 73 \end{array}$$

EXAMPLE THIRD.

Multiply $23\frac{1}{3}$ by $19\frac{1}{4}$ by business method.

Here $\frac{1}{4}$ of 23 to the nearest unit is 6, and $\frac{1}{3}$ of 19 to the nearest unit is 6; then 6 plus 6 is 12, so we simply say, 19 times 23 are 437 and 12 are 449, Ans.

$$\begin{array}{r} 23\frac{1}{3} \\ 19\frac{1}{4} \\ \hline 449 \end{array}$$

N. B. In multiplying the whole numbers together always use the single-line method.

EXAMPLE FOURTH.

Multiply $128\frac{2}{3}$ by 25 by business method.

Here $\frac{2}{3}$ of 25 to the nearest unit is 17, so we simply say, 25 times 128 are 3200 and 17 are 3217, the answer.

$$\begin{array}{r} 128\frac{2}{3} \\ 25 \\ \hline 3217 \end{array}$$

PRACTICAL EXAMPLES FOR BUSINESS MEN.

1. What is the cost of $17\frac{1}{2}$ lbs. sugar at $18\frac{3}{4}$ cts. per lb.?

Here $\frac{3}{4}$ of 17 to the nearest unit is 13, and $\frac{1}{2}$ of 18; is 9 13 plus 9 is 22, so we simply say, 18 times 17 are 306 and 22 are 328, the answer.

$$\begin{array}{r} 17\frac{1}{2} \\ 18\frac{3}{4} \\ \hline \$3.28 \end{array}$$

2. What is the cost of 11 lbs. 5 oz. of butter at $33\frac{1}{3}$ cts. per lb.?

Here $\frac{1}{3}$ of 11 to the nearest unit is 4, and $\frac{5}{16}$ of 33 to the nearest unit is 10; then 4 plus 10 is 14, so we simply say, 33 times 11 are 363, and 14 are 377, Ans.

$$\begin{array}{r} 11\frac{5}{16} \\ 33\frac{1}{3} \\ \hline \$3.77 \end{array}$$

3. What is the cost of 17 doz. and 9 eggs at $12\frac{1}{2}$ cts. per doz.?

Here $\frac{1}{2}$ of 17 to the nearest unit is 9, and $\frac{9}{12}$ of 12 is 9; then nine plus 9 is 18, so we simply say, 12 times 17 are 204 and 18 are 222, the answer.

$$\begin{array}{r} 17\frac{9}{12} \\ 12\frac{1}{2} \\ \hline \$2.22 \end{array}$$

4. What will be the cost of $15\frac{3}{4}$ yds. calico at $12\frac{1}{2}$ cts. per yd.? Ans. \$1.97.

N. B. To multiply by aliquot parts of 100, see page 44

RAPID PROCESS OF MARKING GOODS.

A VALUABLE HINT TO MERCHANTS AND ALL RETAIL DEALERS IN FOREIGN AND DOMESTIC DRY GOODS.

RETAIL merchants, in buying goods by wholesale, buy a great many articles by the dozen, such as boots and shoes, hats and caps, and notions of various kinds. Now, the merchant, in buying, for instance, a dozen hats, knows exactly what one of those hats will retail for in the market where he deals; and, unless he is a good accountant, it will often take him some time to determine whether he can afford to purchase the dozen hats and make a living profit in selling them by the single hat; and in buying his goods by auction, as the merchant often does, he has not time to make the calculation before the goods are cried off. He therefore loses the chance of making good bargains by being afraid to bid at random, or if he bids, and the goods are cried off, he may have made a poor bargain, by bidding thus at a venture. It then becomes a useful and practical problem to determine *instantly* what per cent. he would gain if he retailed the hats at a certain price.

To tell what an article should retail for to make a profit of 20 per cent.,

RULE.—*Divide what the articles cost per dozen by* 10, *which is done by removing the decimal point one place to the left.*

For instance, if hats cost \$17.50 per dozen, remove the decimal point one place to the left, making \$1.75, what they should be sold for apiece to gain 20 per cent. on the cost. If they cost \$31.00 per dozen, they should be sold at \$3.10 apiece, etc. We take 20 per cent. as the basis for the following reasons, viz.: because we can determine instantly, by simply removing the decimal point, without changing a figure; and, if the goods would not bring at least 20 per cent. profit in the home market, the merchant could not afford to purchase and would look for goods at lower figures.

Reason.—The reason for the above rule is obvious: For if we divide the cost of a dozen by 12, we have the cost of a single article; then if we wish to make 20 per cent. on the cost, (cost being $\frac{1}{1}$ or $\frac{5}{5}$,), we add the 20 per cent., which is $\frac{1}{5}$, to the $\frac{5}{5}$, making $\frac{6}{5}$ or $\frac{12}{10}$; then as we multiply the cost, divided by 12, by the $\frac{12}{10}$ to find at what price one must be sold to gain 20 per cent., it is evident that the 12s will cancel, and leave the cost of a dozen to be divided by 10, which is done by removing the decimal point one place to the left.

1. If I buy 2 doz. caps at \$7.50 per doz., what shall I retail them at to make 20%? Ans. 75 cts.

2. When a merchant retails a vest at \$4.50 and makes 20%, what did he pay per doz.? Ans. \$45.

3. At what price should I retail a pair of boots that cost \$85 per doz., to make 20%? Ans. \$8.50.

RAPID PROCESS OF MARKING GOODS AT DIFFERENT PER CENTS.

Now, as removing the decimal point one place to the left, on the cost of a dozen articles, gives the selling price of a single one with 20 per cent. added to the cost, and, as the cost of any article is 100 per cent., it is obvious that the selling price would be 20 per cent. more, or 120 per cent.; hence, to find 50 per cent. profit, which would make the selling price 150 per cent., we would first find 120 per cent., then add 30 per cent., by increasing it one-fourth itself; to make 40 per cent., add 20 per cent., by increasing it one-sixth itself; for 35 per cent., increase it one-eighth itself, etc. Hence, to mark an article at any per cent. profit, we have the following

GENERAL RULE

First find 20 *per cent. profit, by removing the decimal point one place to the left on the price the articles cost a dozen; then, as* 20 *per cent. profit is* 120 *per cent., add to or subtract from this amount the fractional part that the required per cent. added to* 100 *is more or less than* 120.

Merchants, in marking goods, generally take a per cent. that is an aliqot part of 100, as 25%, $33\frac{1}{3}$%, 50%, etc. The reason they do this is because it makes it much easier to add such a per cent. to the cost; for instance, a merchant could

mark almost a dozen articles at 50 per cent. profit in the time it would take him to mark a single one at 49 per cent. For the benefit of the student, and for the convenience of business men in marking goods, we have arranged the following table:

TABLE

For Marking all Articles bought by the Dozen.

N. B. Most of these are used in business.

To	make	20%	remove	the point	one place	to the left.			
"	"	80%	"	"	"	and	add	$\frac{1}{2}$	itself.
"	"	60%	"	"	"	"	"	$\frac{1}{3}$	"
"	"	50%	"	"	"	"	"	$\frac{1}{4}$	"
"	"	44%	"	"	"	"	"	$\frac{1}{5}$	"
"	"	40%	"	"	"	"	"	$\frac{1}{6}$	"
"	"	$37\frac{1}{2}$%	"	"	"	"	"	$\frac{1}{7}$	"
"	"	35%	"	"	"	"	"	$\frac{1}{8}$	"
"	"	$33\frac{1}{3}$%	"	"	"	"	"	$\frac{1}{9}$	"
"	"	32%	"	"	"	"	"	$\frac{1}{10}$	"
"	"	30%	"	"	"	"	"	$\frac{1}{12}$	"
"	"	28%	"	"	"	"	"	$\frac{1}{15}$	"
"	"	26%	"	"	"	"	"	$\frac{1}{20}$	"
"	"	25%	"	"	"	"	"	$\frac{1}{24}$	"
"	"	$12\frac{1}{2}$%	"	"	"	subtract		$\frac{1}{16}$	"
"	"	$16\frac{2}{3}$%	"	"	"	"		$\frac{1}{36}$	"
"	"	$18\frac{3}{4}$%	"	"	"	"		$\frac{1}{96}$	"

If I buy 1 doz. shirts for \$28.00, what shall I retail them for to make 50%? Ans. \$3.50.

EXPLANATION.—Remove the point one place to the left, and add on $\frac{1}{4}$ itself.

Where the Multiplier is an Aliquot Part of 100.

Merchants in selling goods generally make the price of an article some aliquot part of 100, as in selling sugar at $12\frac{1}{2}$ cents a pound or 8 pounds for 1 dollar, or in selling calico for $16\frac{2}{3}$ cents a yard or 6 yards for 1 dollar, etc. And to become familiar with all the aliquot parts of 100, so that you can apply them readily when occasion requires, is perhaps the most useful, and, at the same time, one of the easiest arrived at of all the computations the accountant must perform in the practical calculations of the counting-room.

TABLE OF THE ALIQUOT PARTS OF 100 AND 1000

N. B. Most of these are used in business.

$12\frac{1}{2}$ is $\frac{1}{8}$ part of 100.	$8\frac{1}{3}$ is $\frac{1}{12}$ part of 100
25 is $\frac{2}{8}$ or $\frac{1}{4}$ of 100.	$16\frac{2}{3}$ is $\frac{2}{12}$ or $\frac{1}{6}$ of 100
$37\frac{1}{2}$ is $\frac{3}{8}$ part of 100.	$33\frac{1}{3}$ is $\frac{4}{12}$ or $\frac{1}{3}$ of 100.
50 is $\frac{4}{8}$ or $\frac{1}{2}$ of 100.	$66\frac{2}{3}$ is $\frac{8}{12}$ or $\frac{2}{3}$ of 100.
$62\frac{1}{2}$ is $\frac{5}{8}$ part of 100.	$83\frac{1}{3}$ is $\frac{10}{12}$ or $\frac{5}{6}$ of 100.
75 is $\frac{6}{8}$ or $\frac{3}{4}$ of 100.	125 is $\frac{1}{8}$ part of 1000.
$87\frac{1}{2}$ is $\frac{7}{8}$ part of 100.	250 is $\frac{2}{8}$ or $\frac{1}{4}$ of 1000.
$6\frac{1}{4}$ is $\frac{1}{16}$ part of 100.	375 is $\frac{3}{8}$ part of 1000.
$18\frac{3}{4}$ is $\frac{3}{16}$ part of 100.	625 is $\frac{5}{8}$ part of 1000.
$31\frac{1}{4}$ is $\frac{5}{16}$ part of 100.	875 is $\frac{7}{8}$ part of 1000.

To multiply by an aliquot part of 100,

RULE.—*Add two ciphers to the multiplicand, then take such part of it as the multiplier is part of* 100,

N. B. If the multiplicand is a mixed number reduce the fraction to a decimal of two places before dividing.

3. To multiply any number by 125 add three ciphers, and divide by 8.

Multiply 3467 by 125. Product, 433375.

8)3467000

433375

NOTE.—By annexing three ciphers the number is increased one thousand times; and by dividing by 8, the quotient will be only one-eighth of 1000, that is 125 times.

4. To multiply any number by $16\frac{2}{3}$ add two ciphers, and divide by 6.

Multiply 3768 by $16\frac{2}{3}$. Product, 62800.

6)376800

62800

5. To multiply any number by $166\frac{2}{3}$ add three ciphers, and divide by 6.

Multiply 7875 by $166\frac{2}{3}$. Product, 1312500.

6)7875000

1312500

6. To multiply any number by $33\frac{1}{3}$ add two ciphers, and divide by 3.

Multiply 9879 by $33\frac{1}{3}$. Product, 329300

)987900

329300

RATIONALE.—As in the last case, by annexing two ciphers, we increase the multiplicand one hundred times; and by dividing the number by 3, we only increase the multiplicand thirty-three and one-third times, because $33\frac{1}{3}$ is one-third of 100.

7. To multiply any number by $333\frac{1}{3}$ add three ciphers, and divide by 3.

Multiply 4797 by $333\frac{1}{3}$. Product, 1599000.

$$\begin{array}{r} 3)\underline{4797000} \\ 1599000 \end{array}$$

8. To multiply any number by $6\frac{2}{3}$ add two ciphers, and divide by 15; or add one cipher and multiply by $\frac{2}{3}$.

Multiply 1566 by $6\frac{2}{3}$.

$$\begin{array}{rl} 15)\underline{156600} & \\ 10440 & \text{First method.} \end{array}$$

$$\begin{array}{rl} 15660 & \\ \underline{2} & \\ 3)\underline{31320} & \\ 10440 & \text{Second method.} \end{array}$$

9. To multiply any number by $66\frac{2}{3}$ add three ciphers, and divide by 15; or add two ciphers and multiply by $\frac{2}{3}$.

Multiply 3663 by $66\frac{2}{3}$.

15)3663000

244200 First method.

366300
2

3)732600

244200 Second method.

10. To multiply any number by $8\frac{1}{3}$ add two ciphers, and divide by 12.

Multiply 2889 by $8\frac{1}{3}$. Product, 24075.

12)288900

24075

11. To multiply any number by $83\frac{1}{3}$ add three ciphers, and divide by 12.

Multiply 7695 by $83\frac{1}{3}$. Product, 641250.

12)7695000

641250

12. To multiply any number by $6\frac{1}{4}$ add two ciphers, and divide by 16 or its factors—4×4.

Multiply 7696 by $6\frac{1}{4}$. Product, 48100

4)769600

4)192400

48100

13. To multiply any number by $62\frac{1}{2}$ add three ciphers, and divide by 16 or its factors—4×4.

Multiply 3264 by $62\frac{1}{2}$. Product, 204000.

$$\begin{array}{r} 16)\underline{3264000} \\ 204000 \end{array}$$

14. To multiply any number by $18\frac{3}{4}$, add two ciphers, and multiply by 3, and divide by 16 or its factors—4×4.

Multiply 768400 by $18\frac{3}{4}$. Product, 144075.

$$\begin{array}{r} 768400 \\ \underline{3} \\ 16)\underline{2305200} \\ 144075 \end{array}$$

15. To multiply any number by $31\frac{1}{4}$ add three ciphers, and divide by 32 or its factors—4×8.

Multiply 7847 by $31\frac{1}{4}$. Product, $245218\frac{6}{8}$.

$$\begin{array}{r} 4)\underline{7847000} \\ 8)\underline{1961750} \\ 245218\frac{6}{8} \end{array}$$

16. To multiply any number by $37\frac{1}{2}$, add two ciphers and multiply by 3, and divide by 8.

Multiply 2976 by $37\frac{1}{2}$. Product, 111600.

```
  297600
       3
  ------
8)892800
  ------
  111600
```

17. To multiply any number by $87\frac{1}{2}$ add two ciphers, divide by 8, and subtract the quotient.

Multiply 6768 by $87\frac{1}{2}$.

```
8)676800
   84600
  ------
  592200 Ans.
```

18. To multiply any number by 75 add two ciphers, divide by 4, and subtract the quotient.

Multiply 4968 by 75.

```
4)496800
  124200
  ------
  372600 Ans.
```

19. To multiply by 99 add two ciphers, and subtract the given number from the result.

Multiply 31416 by 99.

```
          3141600
            31416
          -------
Product,  3110184
```

As 99 times is 1 time less than 100 times, and

adding 00 is in effect multiplying by 100, one time the multiplicand is deducted, which leaves 99 times, as required.

This principle is applicable where any number of 9's is the multiplier; as many ciphers being added of course as there are 9's. If the multiplier were 98, we would subtract twice the multiplicand; if 97, three times, and so on.

20. To multiply by any number of 9's.

RULE.—*Annex as many ciphers to the multiplicand as there are 9's in the multiplier, and from this number subtract the number to be multiplied, and the remainder is the product required.*

NOTE.—To multiply by any number of 3's, proceed as above, and divide the product by 3; but if it be required to multiply by 6's, proceed as above, and then multiply the product by two, and divide the result by 3, and the quotient is the product.

21. To multiply by any number between 10 and 20, as 16 or 18, multiply by the units' figure, and set the product under the multiplicand; but put it one place to the right; then add the lines together. The reason is evident on looking at the calculation.

Multiply 3854 by 16.

```
 3854
 23124
 -----
 61664 Ans.
```

On the same principle, if any number of ciphers intervene, as 106, 1006, 10006, etc., set the product so many places farther to the right.

Multiply 3854 by 1006.

```
3854
  23124
-------
3877124 Ans.
```

Cross Multiplication is a mode of multiplying by large multipliers in a single line; and by practice the operation may be performed with great expedition. It is necessary to begin with small numbers, say of two places, and carry the calf diligently, if you would carry the ox successfully.

Here we multiply 5×2 and set down and carry as usual; then to what you carry add 5×3 and 4×2, which gives 24; set down 4 and carry 2 to 4×3, which gives 14. It is obvious that this is just the usual mode, with the intermediate work done in the head.

```
  32
  45
----
1440
```

Here the first and second places are found as before; for the third, add 6×1, 4×3, 5×2, with the 2 you had to carry, making 30; set down 0 and carry 3; then drop the units' place and multiply the hundreds and tens crosswise, as you did the tens and units, and you find the thousands' figure; then,

```
  123
  456
-----
56088
```

dropping both units and tens, multiply the 4×1, adding the 1 you carried, and you have 5, which completes the product. The same principle may be extended to any number of places; but let each step be made perfectly familiar before advancing to another. Begin with two places, then take three, then four, but always practicing some time on each number, for any hesitation as you progress will confuse you.

To Multiply by 21, 31, etc., to 91 in a single line, multiply by the tens' figure and set the product one place to the left underneath the multiplicand; then add.

Multiply 3854 by 21.

```
  3854
 7708
 -----
 80934 Ans.
```

If ciphers intervene, as 201, 3001, etc., multiply as before, but set the product as many additional places to the left as there are ciphers.

Multiply 3854 by 6001.

```
    3854
 23134
 -------
 2316254 Ans.
```

The following is a convenient mode of multi-

plying by any two figures, and is not difficult to apply:

Multiply 3754
By 27

Product, 101358

I here multiply 27 by 4, setting down the first product figure and carrying the others; I then multiply by 5 and set down and carry in the same way—so proceeding to the highest place of the multiplicand.

Where the multiplier is not too large, and can be divided into two or more factors, there is a saving in adopting the following mode, and it may be used as a proof of the common mode. If I seek to multiply, say 7864 by 24, it requires in the usual way two product lines, and finding their sum by addition, making a third operation; but if I multiply by 6, and that product by 4, or by 8 and 3, or 12 and 2, the business is dispatched in two lines. But being able to multiply in a single line is still better.

The same remarks apply in Division, and hence there is often economy of figures in dividing by factors of your divisor; and if a remainder occurs only in your first division it is the true one; but if in the second only, then multiply such remainder by the first divisor, or all if more than one;

and if there was a remainder on the first division also, it must be added in, and the sum will be the true remainder.

Divide 73640 by 24.

6)73640

4)12273 and 2 over.

3068 and 1 over. $1 \times 6 + 2 = 8$, the true rem'der.

Or— 4)73640

6)18410

3068 and 2 over, and $2 \times 4 = 8$, as before.

Another mode of proving Division is to divide the dividend by the quotient, and the result will be the divisor—the same remainder occurring in one case as in the other, but not of the same fractional value; for as the dividend exceeds some multiple of the divisor and quotient, just the amount of the remainder, that remainder will be the same, without regard to which of the factors occupies the divisor's place; and as the divisors would differ, the value of the fraction formed by the remainder and divisor must differ. To make the dividend an exact multiple, subtract the remainder from it.

```
Multiply 3756
      By  182
         -----
          7512
         30048
         3756
         ------
        683592
```

In the above example the second product line may be conveniently found by multiplying the first by 4; but if there is an error in the first it will run into the second. A shorter mode would be to multiply the first product line by 9, which would give the product by 18 in one line, and save work. Try it.

As a large number of the problems solved in this book are solved by different modes, they furnish a great variety of modes of proof.

To multiply a decimal or a mixed number by 10, 100, 1000, etc., remove the decimal point one, two, or three places to the right; and to divide by such numbers remove the point as many places to the left; and if there be not a sufficient number of places on the left, ciphers must be prefixed.

To multiply or divide by any composite number; multiply, or divide, as the case may be, by the factors of the numbers successively.

This was sufficiently illustrated under the head of Proofs.

The French mode of operating in Long Division has some advantage over ours. They place the divisor on the right of the dividend, as we do the quotient, and place the quotient underneath the divisor, by which the figures to be multiplied together are brought near each other. Thus:

```
Divid.  Divis.
 3936)96
 384  41 Quotient.
 ----
   96
   96
   --
```

For sake of brevity they frequently omit the product figures, setting down only the remainders, which they find as they pass along. Thus:

```
3936)96
  96 41, Ans.
  --
```

This, however, applies to our mode as well as theirs.

The following mode of multiplying by large multipliers in a single line, is both curious and useful:

Multiply 7865 by 432 in a single line.

On a slip of paper, separate from that on which the multiplicand is written, place the multiplier in inverted order: thus, 234 and close to the upper edge of the paper. Then bring the multiplier so that the 2 shall be directly under the 5, or units'

place of the multiplicand: multiply those figures, set down 0 and carry 1. Slide the paper to the left one place, that 2 may be under 6, and 3 under 5; and to the 1 you carried add the products of the 2 by 6, and 3 by 5, making 28—set down 8 and carry 2. Again move your paper one place to the left, and to the 2 you carried, add the several products of the multiplicand figures with the figures of the multiplier that are under them, viz: 8×2, 6×3, 5×4, and the result will be 56; set down 6 and carry 5. Slide again and you have 5 (that you carried) $+14+24+24=67$. Thus proceed toward the left until the multiplier passes from under the multiplicand, each time adding what you carry, to the several products of the figures that stand one over the other, the result will be 3397680. These additions will soon be performed at a glance, as the products are obvious *without the formality of naming the factors.*

To understand these directions clearly, factors must be placed on slips of paper, and the directions strictly complied with; by which the mode of operation and the reason will be better understood in ten minutes, than in three hours without them. When familiar with the slide, the operator may proceed without it, and perform operations astonishing to the uninitiated; the largest numbers being multiplied together readily in a single line.

ON THE PROPERTIES OF NUMBERS.

PROPOSITION 1.

Digits in our system of notation increase in value from right to left in a tenfold ratio.

PROPOSITION 2.

In any series of digits expressing a number, the value of any digit is greater than the value of all the digits on its right.

This property results also from value according to place; and that the proposition is true is obvious, for if we take the smallest digit (1) and place it on the left of the largest (9) we form 19; the 1 expresses 10 units, while the 9 expresses but 9 units; and let us add what numbers of nine we may, the unit will constantly retain its greater value: *e. g.* 19, 199, 1999, etc. Not only is the left hand digit higher in value than all upon its right, but the same remark applies to each digit, in reference to those on its right.

PROPOSITION 3.

If the sum of the digits in any number be a multiple of 9, the whole number is a multiple of 9.

This is one of several peculiar properties of the number 9, all arising from its being just one less than the radix of our system of notation, and

hence the highest number expressed by a single character; and these properties will belong to the highest number so expressed in any system. We might go a step further in reference to this property, and say that it belongs to any number that will divide the radix of the system, and leave one as a remainder.

If we carefully examine the genesis of numbers, we must see that so far as the number 9 is concerned, this is an accidental property, resulting from our scale of notation. We constantly express each successive number from unity to 9, inclusive, by a digit of greater value than any preceeding it; but when we pass 9 we express the next number, 10, by a unit and a cipher. The number is one greater than 9, and the sum of its digits is 1. Eleven is 2 greater, and the sum of its digits is $1+1=2$. Thus we proceed, the sum of the digits constantly expressing the excess over 9, until we reach 18, or twice 9. Nineteen is 1 and 9, and it is one over twice 9. 20 is 2 over twice 9, and the sum of its digits is 2. The same course continued to millions, would but produce the same recurring result. Nine is 1 less than 10; twice 9 are 2 less than 20; 3×9 are 3 less than 30, and so on; and hence the 1 of 10, 2 of 20, 3 of 30 etc., come just in the proper place to keep up the excess above 9 and its multiples. If the multiples

of 9 did not constantly fall at each product, one further behind the corresponding multiples of 10, the two of 20, 3 of 30, etc., would not fall in the right place, to keep up the regular order of the series.

PROPOSITION 4.

If the sum of the digits in any number be a multiple of 3, *the number is a multiple of* 3.

The same reasoning applied to the number 9 to show the correctness of the preceding proposition, will show the correctness of this. Ten, the sum of whose digits is 1, is 1 over 3 times 3; 11, the sum of whose digit is 2, is 2 more than 3 times 3; 12, the sum of whose digit is 3, is a multiple, etc., etc.

PROPOSITION 5.

Dividing any number by 9 *or* 3, *will leave the same remainder as dividing the sum of its digits by* 9 *or* 3.

This proposition follows as a matter of course from the two next preceding it; and we shall adduce no other proof of its correctness. Like the former, it is an accidental property of the highest number expressed by a single digit in any system, and of all its factors. If 9 were the basis of our system, these properties would belong to 8, 4, and 2; if 8, then 7 only, since 7 has no factors; and

if 7 were the basis, then 6, 3, and 2 would possess these properties; and if 12 were the basis, then 11 only would possess such properties; for it would in that case be expressed by a single digit, and would be the highest number so expressed. Twelve would be written with a unit and a cipher as 10 now is; and 11 being prime, it would be the only number that would divide the radix of the system and leave 1 as a remainder.

As early as 1657, Dr. Wallis, of England, applied this principle to prove the correctness of operations in the elementary rules of Arithmetic, and the practice has been continued to the present time. The operation is performed thus:

Add	79864=7
	32075=8
	83214=0
	61840=1
	48167=8
	305160=6

We cast the nines out of each number separately, and set the excess on the right. We then cast the nines out of the sum total 305160, and also out of the sum of the excesses 8+1+8+7, and they are equal: both being 6, and we hence infer that the work is right. To cast out the nines, the number may be divided by 9; but a better way is to add the digits together in each number, rejecting 9 whenever it occurs, and carrying forward only the excess. Thus 7 and 8 are 15; 9 being rejected, we carry 6 to 6 is 12; rejecting 9, we carry 3 to 4=7; the num-

ber carried in each place is the excess over 9; and where 9 occurs it is passed over.

In Subtraction cast out the nines from the minuend and subtrahend, and also from the remainder If the excess in the remainder is equal to the difference of excesses in the minuend and subtrahend, the work is right.

Here, as we can not take 8 from 6, we take from 9 and add 6; the result, 7 agrees with the excess above 9 in the difference of the numbers.

From	6894321=6
Take	2960864=8
Leaves	3933457=7

In Multiplication, find the excess in the factors, and if the excess in the product of these two excesses equals the excess in the product of the factors the operation is correct.

Multiply	48756=3
By	245=2
	243780
	195024
	97512
	11945220=6

3	2
6	6

This is often called proving by the cross; and instead of placing the excesses after marks of equality, they are placed in the angles of a cross as on the right hand of the above operation.

In Division cast the nines out of the divisor, dividend, quotient, and remainder; then to the product of the excesses in the divisor and quotient, add the excess in the remainder, and cast the nines out of the sum, and if the excess equal that in the dividend the work is right.

	Excess in Divisor	0
	Excess in Quotient	8
27)465		
	Product of Excess	0
17+6	Add Excess of Remainder	6
	Excess in Dividend	6=6

Hence the work is right, the excesses being equal.

It is proper to remark that this mode of proof is liable to much objection. If the figures become transposed, or if mistakes are made that balance each other, the work will prove right when it is wrong. The work will, however, never prove wrong when it is right. In the product above, you may transpose the digits as you please, the work will prove, since the excess is the same whatever is the order of the digits; and ciphers may always be omitted. Or if mistakes balance each other, as if instead of 91145 it be 83216. The excess here will be the same and the work will prove, though not a figure is right.

PROPOSITION 6.

If from any number the sum of its digits be subtracted, the remainder is a multiple of 9.

F'm 31416	*For,*
Take 15=3+1+4+1+6	31416÷9=3490+6
	15+9= 1+6
9)31401	
	31401÷9=3489
3489 times 9, diff.	

As the remainder on dividing the given number by 9 will be just the same as on dividing the sum of its digits, (Prop. 5,) it is obvious that the difference must be an even multiple. The given number is 6 more than 3490 times 9; the sum of digits is 6 more than 1 time, hence their difference is 3489 times.

This principle is sometimes used as a puzzle; you may let a person write down any number for a minuend, then have the party add the figures and place their sum for a subtrahend; then, after subtracting, let the person rub out any one figure in the remainder and give you the figures left in the remainder. You, by adding the figures given and subtracting their sum from the next multiple of 9, may tell the figure rubbed out, although you did not see a figure that was written. Try it.

$$\begin{array}{r} 6745 \\ 22 \\ \hline 6723 \end{array}$$

PROPOSITION 7.

The difference between a given number and the digits composing such number reversed or any how arranged, is always a multiple of 9.

The difference, for instance, between 7425, and any arrangement you can make of the same figures is a multiple of 9.

From	7425	7425	7425
Take	5247	5724	2457
	9)2178	9)1701	9)4968
	242	189	552

This is based on the same reason as the preceding; for whether you take the sum of the digits or transpose the digits, it is the same in effect. The excess over an even multiple being the same as in the given number, the difference must necessarily be an even multiple.

A practical application is sometimes made of this principle by a person setting down two rows of figures for subtraction, but being careful to have the figures of the subatrahend and minuend the same, though differently arranged. One figure of the remainder is then stricken out, and the puzzlo is to restore it without seeing the minuend and subtrahend It is done by taking such number as will make the remainder a multiple of 9.

Here if 5, 4, or 1, be erased, any one may restore it; but if the 9 or 0 be removed he can not know whether a 9 or a cipher should be supplied, as either will make the number a multiple of 9.

From	7354681
Take	1864537
	5490144

The mode we have adopted in explaining the four last preceding propositions appears to us plain and sufficiently satisfactory.

In addition to the use of these properties as modes of proof, they are the key to many numeral puzzles and amusing questions; and hence the care we have bestowed in explaining the principle. What has been said may be a sufficient explanation of the following article on the "*Wonderful Properties of the Number Nine:*"

"Multiply 9 by itself or any other digit, and the figures of the product added will be 9.

Take the sum of our numerals $1+2+3+4+5+6+7+8+9=45$, the digits of which, $4+5=9$. Multiply each of these digits by 9, and their sum will be 405; which added $4+0+5=9$; and $405\div9=45$, also a multiple of 9.

Multiply any number, large or small, by 9, or 9 times any digit, and the sum of the digits of the product will be a multiple of 9.

Multiply the 9 digits in their order, 1 2 3 4 5 6

7 8 9, by 9, or any multiple of 9 not exceeding 9 times 9, and the product, except the tens' place, will be all the same figures, while the tens' place will be filled with 0. The significant figure will always be the number of times 9 is contained in the multiplier.

27, or 3 times 9, will produce all 3s; 4 times 9 all 4s.

Omit 8 in the multiplicand and the product will be all the same digits, the 0 having disappeared."

$$\begin{array}{r} 123456789 \\ 18 = 9 \times 2 \\ \hline 987554312 \\ 123456789 \\ \hline 2222222202 \\ \hline \end{array}$$

To a superficial observer the above results may seem accidental, but investigation will show that they all flow from the laws and principles we have laid down; and that a much longer list might be made of apparently simple and detached facts, but really of results flowing from well established laws. *There are no unaccountable properties in numbers.*

While the number 9 has some peculiar properties from being the next below the radix of the system, the number 11 has some peculiarities from being next above the radix. Among these are the following:

"If from any number the sum of the digits standing in the *odd* places be subtracted, and to

the remainder the sum of the digits standing in the *even* places be added, then the result is a multiple of 11." Again, "If the sum of the digits standing in the *even* places be equal to the sum of the digits standing in the *odd* places, or differ by 11 or any of its multiples, the number is a multiple of 11."

As these however are of no practical utility, we shall not discuss them.

The number 7 has also some peculiarities, but we shall name only one, as they are useless. *If a number be divided into periods of three figures each, beginning at the units' place, when the difference of the sums of the alternate periods is a multiple of 7, the whole number is a multiple of 7.*

Here 862—428 is a multiple of 7; and so is 907—382; therefore the whole number is a multiple of 7.

```
7)382,907,428,862
  ———————————————
   54,701,061,266
  ———————————————
```

The division of numbers into *Even* and *Odd* seems to arise from considering them in pairs. The following facts growing out of this division will be readily understood:

The sum of two even numbers is even, and so is their difference: 8+4=12; 8—4=4.

The sum of an odd number of odd numbers is odd; but the sum of an even number of odd numbers is even: 3+5+7=15; 3+5=8; and 5+7=12.

An even and an odd number being added together, or one subtracted from the other, the result will be odd: $8+3=11$; $8-3=5$.

If a number has 0, 2, 4, 6 or 8 in the units' place, it is divisible by 2, and is consequently even.

No odd number can be divided by an even number without a remainder.

If an odd number measure an even one, it will also measure the half of it. 7 measures 42, and therefore measures 21, the half of it.

If the sum of the digits standing in the EVEN *places, be equal to the sum of the digits standing in the* ODD *places, then the number is divisible by* 11.

Thus the numbers 121, 363, 12133, 48422, etc., are all divisible by 11.

Multiplication and Division.

To multiply one-half, is to take the multiplicand one-half of *one time;* that is, take one-half of it, or divide it by 2.

To multiply by $\frac{1}{3}$, take a third of the multiplicand, that is, divide it by 3.

To multiply by $\frac{2}{3}$, take $\frac{1}{3}$, first, and multiply that by 2; or, multiply by 2 first, and divide the product by 3.*

*Sometimes one operation is preferable, and sometimes the other; good judgment alone can decide when the case is before us.

EXAMPLES.

1. What will 360 barrels of flour come to at $5\frac{1}{4}$ dollars a barrel. At 1 dollar a barrel it would be 360 dollars; at $5\frac{1}{4}$ dollars, it would be $5\frac{1}{4}$ *times as much.*

	360
	$5\frac{1}{4}$
5 times,	1800
$\frac{1}{4}$ of a time,	90
Ans.	\$1890

Before we attempt to divide by a mixed number, such as $2\frac{1}{2}$, $3\frac{1}{4}$, $5\frac{3}{8}$, etc., we must explain, or rather *observe* the principle of division, namely: *That the quotient will be the same if we multiply the dividend and divisor by the same number.* Thus 24 divided by 8, gives three for a quotient. Now, if we double 24 and 8, or multiply them by any number whatever, and then divide, we shall still have 3 for a quotient. 16)48(3; 32)96(3, etc.

Now, suppose we have 22 to be divided by $5\frac{1}{2}$; we may double both these numbers, and thus be clear of the fraction, and have the same quotient. $5\frac{1}{2}$)22(4 is the same as 11)44(4.

How many times is $1\frac{1}{4}$ contained in 12? *Ans.* Just as many times as 5 is contained in 48. The 5 is 4 times $1\frac{1}{4}$, and 48 is 4 times 12. From these observations, we draw the following rule for dividing by *a mixed number.*

RULE.—*Multiply the whole number by the lower term of the fraction; add the upper term to the product for a divisor; then multiply the dividend by the lower term of the fraction, and then divide.*

How many times is $1\frac{1}{5}$ contained in 36? *Ans.* 30 times.

N. B. If we multiply both these numbers by 5, they will have the same *relation* as before, and a quotient is nothing but a relation between two numbers. After multiplication, the numbers may be considered as having the *denomination of fifths.*

How many times is $\frac{1}{4}$ contained in 12? *Ans.* 48 times.

One-fourth multiplied by 4, gives 1; 12, multiplied by 4, gives 48. Now, 1 in 48 is contained 48 times.

Divide 132 by $2\frac{3}{4}$. *Ans.* 48.

Divide 121 by $15\frac{1}{8}$. *Ans.* 8

How many times is $\frac{3}{4}$ contained in 3? *Ans.* 4 times.

By a little attention to the relation of numbers, we may often contract operations in multiplication. A dead uniformity of operation *in all cases* indicates a mechanical and not a scientific knowledge of numbers. As a uniform principle, it is much easier to multiply by the small numbers, 2, 3, 4, 5, than by 7, 8, 9.

Multiply	4532	Multiply	4532
by	639	by	963
(63=9×7.)	40788		40788
	285516		285516
Product,	2895948	Product,	4364316

In both the foregoing examples we multiply the product of 9 by 7, because 7 times 9 are equal to 63.

Because 9 is in the place of hundreds in example 2, the product for the other two figures is set two places toward the right.

In this last example we may commence with the 3 units in the usual way; then *that product* by 2, because 2 times 3 are 6; then the product of 3 by 3, which will give the same as the multiplicand by 9. The *appearance* of the work would then be the same as by the usual method, but would be easier, as we actually multiply by smaller numbers.

Multiply	40788	
by	497	
	285516	Product of the 7 units.
	1998612	As 7×7=49, multiply the product of 7 by 7.
	20271636	

Every fact of this kind, though extremely simple, should be known by all who seek for knowledge in figures.

Multiply 785460
by 14412

First multiply by 12, then that product by 12.

9425520
113106240

11320049520

Multiply 576
by 186

(6×3 18.) 3456
10368

107136

Multiply this last number, 3456, (which is 6 times 576,) by 3, and place the product in the place of tens, and we have 180 times 576. Observe the same principle in the following examples:

Multiply 576
by 618

Commence with 6. 3456
(6×3=18.) 10368

355968

Multiply 40788
by 497

(7×7=49.) 285516
1998612

20271636

Multiply 61524
by 7209

553716
4429728

Product, 443646516

Multiply this product of 9 by 8, because 9 times 8 are 72, and place the product in the place of 100, because it is 7200.

Multiply	1243	
by	636	
	7458	First by 600.
	44748	Multiply 7458 by 6.
Product,	790548	

Multiply	7864
by	246

This may be done by commencing with the 2; then that product by 2 and 3; or we may commence with the 6 units, and then that product by 4; because 4 times 6 are 24.

Multiply 3764 by 199.

Take 3764 200 times, and from that product subtract 3764.

Multiply 764 by $498\frac{1}{2}$.

Take 764 500 times, and from that product subtract $1\frac{1}{2}$ times 764.

Multiply 396 by $21\frac{3}{4}$, or, (which is the same,) $99 \times 87 = 8700 - 86 \quad 8613$.

N. B.—Ninety-nine is $\frac{1}{4}$ of 396, and 87 is 4 times $21\frac{3}{4}$.

How many times is 125 contained in 2125?

Same as 250 in 4250;
Same as 25 in 425.
Same as 50 in 850;
Same as 5 in 85;
Same as 10 in 170; that is, 17 times.

The object of these changes is to give the learner an accurate and complete knowledge of numbers and of division; and the result is not the only object sought for, as many young learners suppose.

How many times is 75 contained in 575? or divide 575 by 75. *Ans.* $7\frac{2}{3}$.

Divide 800 by $12\frac{1}{2}$. *Quotient*, 64.

Divide 27 by $16\frac{2}{3}$. *Quo.* $1\frac{62}{100}$, or $1\frac{31}{50}$.

A person spent 6 dollars for oranges, at $6\frac{1}{4}$ cents a-piece; how many did he purchase? *Ans.* 96.

When two or more numbers are to be multiplied together, and one or more of them having a cipher on the right, as 24 by 20, we may take the cipher from one number and annex it to the other without affecting the product; thus, 24×20 is the same as 240×2; $286\times1300=28600\times13$; and $350\times70\times40=35\times7\times4\times1000$, etc.

Every fact of this kind, though extremely simple, will be very useful to those who wish to be skillful in operation.

NOTE.—If there are ciphers at the right hand either of the multiplier or multiplicand, or of both, they may be neglected to the close of the operation, when they must be annexed to the product.

REMARKS.—We now give a few examples, for the purpose of teaching the pupil how to use his judgment; he will then have learned a rule *more valuable* than all others.

Multiplication and Division Combined.

WHEN it becomes necessary to multiply two or more numbers together, and divide by a third, or by a product of a third and fourth, it must be *literally done if the numbers are prime.*

For example: Multiply 19 by 13 and divide that product by 7.

This must be done at full length, because the numbers are *prime;* and in all such cases there will result a *fraction.*

But in *actual business* the problems are *almost all* reduceable by short operations; as the prices of articles, or amount called for, always corresponds with some *aliquot* part of our scale of computation. And when two or more of the numbers are *composite numbers,* the work *can always* be contracted.

Example: Multiply 375 by 7, and divide that product by 21. To obtain the answer, it is sufficient to divide 375 by 3, which gives 125.

The 7 divides the 21, and the factor 3 remains for a divisor. Here it becomes necessary to lay down a *plan of operation.*

Draw a perpendicular line and place all numbers that are to be multiplied together under each other, on the right hand side, and all numbers that are divisors under each other, on the left hand side.

EXAMPLES.

Multiply 140 by 36, and divide that product by 84. We place the numbers thus:

$$\begin{array}{r|l} 84 & 140 \\ & 36 \end{array}$$

We may cast out *equal factors* from each side of the line without *affecting the result.* In this case 12 will divide 84 and 36; then the numbers will stand thus:

$$\begin{array}{r|l} 7 & 140 \\ & 3 \end{array}$$

But 7 divides 140, and gives 20, which, multiplied by 3, gives 60 for the result.

Multiply 4783 by 39, and divide that product by 13.

$$\begin{array}{r|l} \cancel{13} & 4783 \\ & \cancel{39}\ 3 \end{array}$$

Three times 4783 must be the result.

Multiply 80 by 9, that product by 21, and divide the whole by the product of $60\times6\times14$.

$$\begin{array}{r|l} 3\ \cancel{60} & \cancel{80}\ 4 \\ 6 & 9 \\ 2\ \cancel{14} & \cancel{21}\ 3 \end{array}$$

In the above divide 60 and 80 by 20, and 14 and 21 by 7, and those numbers will stand canceled as above, with 3 and 4, 2 and 3, at their sides.

Now, the product $3\times6\times2$, on the divisor side, is equal to 4 times 9 on the other, and the remaining 3 is the result.

General Rules for Cancellation.

RULE 1ST. Draw a perpendicular line; observe this line represents the sign of equality. On the right hand side of this line place dividends only; on the left hand side place divisors only; having placed dividends on the right and divisors on the left, as above directed.

2d. Notice whether there are ciphers both on the right and left of the line; if so, erase an equal number from each side.

3d. Notice whether the same number stands both on the right and left of the line; if so, erase them both.

4th. Notice again if any number on either side of the line will divide any number on the opposite side without a remainder; if so, divide and erase the two numbers, retaining the quotient figure on the side of the larger number.

5th. See if any two numbers, one on each side, can be divided by any assumed number without a remainder; if so, divide them by that number, and retain only their quotients. Proceed in the same manner, as far as practicable, then,

6th. Multiply all the numbers remaining on the right hand side of the line for a dividend, and those remaining on the left for a divisor.

7th Divide, and the quotient is the answer.

NOTE.—If only one number remain on either side of the line, that number is the dividend or divisor, according as it stands on the right or left of the line The figure 1 is nct regarded in the operation, because it avails nothing, either to multiply or divide by.

REMARKS.—This method may not work a great many problems, as they are found in some books, but it will work 90 out of every 100 that *ought* to be found in books.

In a book we might find a problem like this:

What is the cost of 2lb. 7oz. 13pwt. of tea, at 7s. 5d. per pound. But the person who should go to a store and call for 3lb. 7oz. and 13pwt. of tea would be a fit subject for a mad-house. The above problem requires downright drudgery, which every one ought to be able to perform; but such drudgery *never occurs in business.*

INTEREST, DISCOUNT, AND AVERAGE.

BEFORE entering upon an investigation of the different modes of calculating interest, it may be interesting to bestow some attention upon the history of the subject, that we may be better prepared to anderstand it.

Among the Jews a law existed that they should not take interest of their brethren. though they were permitted to take it of foreigners. "Thou shalt not lend upon usury to thy brother: usury of money, usury of victuals, usury of any thing that is lent upon usury; unto a stranger thou mayest lend upon usury; but unto thy brother thou shalt not lend upon usury." (Deuteronomy xxiii, 19, 20.) After the dispersion of the Jews they wandered through the earth, but they yet remain a distinct people, mixing, but not becoming assimilated with the people among whom they reside. Still looking to the period when they shall return to the promised land, they seldom engage in permanent business, but pursue traffic, and especially dealing in money; and if their national policy forbids their taking interest of each other, they show no backwardness in taking it unsparingly of the rest of mankind. For ages they have been the money lenders of Europe, and we may safely attribute to this circumstance the prejudice, in some measure, that still exists even in our own country against such as pursue this business as a profession. The prejudice of the Christian against the Jew has been transferred to his occupation, and from the days of Shakspeare, who painted the inexorable Shylock contending for his pound of flesh, down to the present time, the grasping money lender, no

less than the grinding dealer in other matters, has been sneeringly called a Jew.

For ages the taking of any compensation whatever for the use of money was called usury, and was denounced as unchristian; and we find Aristotle, the heathen philosopher, gravely contending that as money could not beget money, it was barren, and usury should not be charged for its use. The philosopher forgot that with money the borrower could add to his flocks and his fields, and profit by the produce of both.

Definition of Terms.

Interest is premium paid for the use of money, goods, or property.

It is computed by per centage—a certain per cent. on the money being paid for its use for a stated time. The money on which interest is paid is called the PRINCIPAL.

The per cent. paid is called the RATE; the principal and interest added together is called the AMOUNT.

When a rate per cent. is stated, without the mention of any term of time, the time is understood to be 1 year.

The first important step in the calculation of simple interest is the arranging of the time for which it is computed. The student must study the

following Propositions carefully, if he would be expert in this important and useful branch of business calculations:

PROPOSITION 1.

If the time consists of years, multiply the principal by the rate per cent., and that product by the number of years.

EXAMPLE 1.—Find the interest of $75 for 4 years at 6 per cent.

Operation.

$75	The decimal for 6 per cent. is
.06	06. There being two places of
——	decimals in the multiplier, we
4.50	point off two in the product.
4	
——	
$18.00 *Ans.*	

PROPOSITION 2.

If the time consists of years and months, reduce the time to months, and multiply the principal by the rate per cent. and number of months together, and divide the result by 12.

NOTE.—The work can always be abbreviated at 4, 6, 8, 9, 12, and 15 per cent., by canceling the per cent., or time, or principal, with the common divisor 12.

EXAMPLE 2.—Find the interest of $240 for 2 years and 7 months at 8 per cent.

First method.

Principal,	$240
Per cent.,	.08
In. for 1yr.,	19.20
2yrs.+7mos.,	31mos.
	12)595.20
	$49.60 *Ans.*

Second method: by cancellation.

$$
\begin{array}{r|ll}
 & \cancel{240}\text{—}20 & \\
\cancel{12} & 8 & \text{rate.} \\
 & 31 & \text{time.} \\
\hline
 & 49.60 & \textit{Ans.}
\end{array}
$$

The operation by canceling is much more brief. We simply place the principal, rate, and time, on the right of the line, and 12 on the left; then we cancel 12 in 240, and the quotient 20 multiplied with 8 and 31 gives the interest at *once.*

NOTE.—After 12 is canceled the product of the remaining numbers is *always* the interest.

PROPOSITION 3.

If the time consists of years, months, and days, reduce the years to months, add in the given months, and place one-third of the days to the right of this number, which we multiply by the principal and rate per cent., and divide by 12, *as before; or cancel and divide by* 12 *before multiplying.*

EXAMPLE 3.—Find the interest of $231 for 1 year, 1 month, and 6 days, at 5 per cent.

First method.

Principal,	$231
Per cent.,	.05
In. for 1yr.,	11.55
1yr.+1mo.+6da.,	13.2mo,
	12)152.460
	$12.705 *Ans.*

Second method: by cancellation.

$$\begin{array}{r|ll} & 231 & \text{prin.} \\ \cancel{12} & 5 & \text{rate.} \\ & \cancel{132}-11 & \end{array}$$

$12.705 *Ans.*

By the second method we cancel 12 in 132, and multiply the quotient 11 by 5 and 231.

NOTE.—When the principal is $, and the time is in years or months, the interest is in cents; if the time is in years, months, and days, the interest is in mills, unless the days are less than 3, in which case it would be in cents, as before.

NOTE.—The reason we divide the days by 3 is because we calculate 30 days for a month, and dividing by 3 reduces the days to the tenth of months.

NOTE.—The three preceding propositions will work any note in interest for any time and at any given rate per cent.

How to Avoid Fractions in Interest.

PROPOSITION 4.

If, when the time consists of years, months, and days, are not divisible by 3, *you can divide the days by* 3, *and annex the mixed number as in Proposition*

3, *or if you wish to avoid fractions, you can reduce the time to interest days, and multiply the principal, rate and days together, and divide the result by* 36 *or its factors*, 4×9.

NOTE.—In this case as in the preceding, the work can almost always be contracted by dividing the rate or time or principal with the divisor 36.

NOTE.—We use the divisor 36, because we calculate 360 interest days to the year. We discard the 0, because it avails nothing to multiply or divide by.

EXAMPLE 4.—Find the interest of $210 for 1 year, 4 months, and 8 days, at 9 per cent.

Year.	Months.	Days.
1	4	8=$16.2\frac{2}{3}$ months or 488 days.

Operation By Prop. 3.	Operation By Prop. 4.
$210	$210
.9	9
18.90	18.90
$16.2\frac{2}{3}$	488
12)307440	36)922320
$25.620 *Ans.*	$25.620 *Ans.*

We will now work the example by cancellation to show its brevity.

Operation by Cancellation.

Time 488 days.

$$\begin{array}{r|l} & 210 \\ \cancel{4}\text{—}\cancel{36} & \cancel{9} \\ & \cancel{488} \quad 122 \end{array}$$

$$\begin{array}{r} 122 \\ 210 \\ \hline \$25.620 \end{array}$$

Now cancel 9 in 36 goes 4 times, then 4 into 488 goes 122. Now multiply remaining numbers together, thus, 210×122 and we have the interest at once.

When the days are not divisible by 3 we reduce the whole time to days; then we place the principal rate and time on the right of the line. Now, because the time is in days, we place 36, on the left of the line for a divisor. (*If the time was months we would place* 12 *on the left.*)

NOTE.—A very short method of reducing time to interest days is to multiply the years by 36; add in 3 times the number of months and the tens' figure of the days, and annex the unit figure; but if the days are less than 10 simply annex them.

EXAMPLE 1.—Reduce 1 year, 2 months, and 6 days, to days.

Years. Months. Days.
$36 \times 1 + 3 \times 2 = 42$ annex $6 = 426$ *Ans.*

EXAMPLE 2.—Reduce 2 years, 3 months and 17 days, to interest days.

Years. M'ths. Days. . Days.

$36\times2+3\times3+1=82$. annex $7=827$ days *Ans.*

NOTE.—The student should commit to memory the multiplication of the number 36 up as far as 9 times 36, and then he can reduce almost instantly years, months, and days, to days.

SIMPLE INTEREST BY CANCELLATION.

RULE.—*Place the principal, time, and rate per cent. on the right hand side of the line. If the time consists of years and months, reduce them to months, and place* 12 *(the number of months in a year) on the left hand side of the line. Should the time consist of months and days, reduce them to days or decimal parts of a month. If reduced to days, place* 36 *on the left. If to decimals parts of a month, place* 12 *only as before.*

Point off two decimal places when the time is in months, and three decimal places when the time is in days.

NOTE. If the principal contains cents, point off four decimal places when the time is in months, and five decimal places when the time is in days.

NOTE.—*We place 36 on the left because there is 360 interest days in a year. (Custom has made this lawful.)*

EXAMPLE 1.—What is the interest on $60 for 117 days at 6 per cent?

Operation.

Here 117×0 must be the answer.	~~6~~0	Both sixes on the right cancels 36 on the left, and we have nothing left to divide by.
~~36~~	~~6~~	
	117	
	$1.170 *Ans.*	

In this case we point off three decimal places because the time is in days. If the time had been 117 months, we would have pointed off but two decimal places.

EXAMPLE 2.—What is the interest of $96.50 for 90 days at 6 per cent?

Operation.

	96.50	9650
~~6~~—~~36~~	~~90~~—-15	15
	~~6~~	
		1.44.750 *Ans.*

Now cancel 6 in 36 and the quotient 6 into 90, and we have no divisor left. Hence 15×96.50 must be the answer.

NOTE—As there are cents in the principal, we point off five decimals; three for days and two for cents. Pay no attention to the decimal point until the clo of the operation.

EXAMPLE 3.—What is the interest of $480 for 361 days at 6 per cent?

$\cancel{6}$—$\cancel{36}$	$\cancel{480}$—80		361
	361		80
	$\cancel{6}$		$28.880 *Ans.*

Now cancel 6 in 36 and the quotient 6 into 480, and we have no divisor left. Hence 80×361 must be the answer.

EXAMPLE 4.—What is the interest of $720 for 9 months at 7 per cent?

$\cancel{12}$	$\cancel{720}$—60		60
	9		9
	7		540
			7
			$37.80 *Ans.*

Now cancel 12 in 720 there is nothing left to divide by. Hence 60×9×7 must be the answer.

N. B. When interest is required on any sum for days only, it is a universal custom to consider 30 days a month, and 12 months a year; and, as the unit of time is a year, the interest of any sum for *one day* is $\frac{1}{360}$, what it would be for a year. For 2 days, $\frac{2}{360}$, etc.; hence if we multiply by the days, we must divide by 360, or divide by 36 and save labor. The old form of this method was to place 360, or 12 and 30, on the left of the line, but using 36 is much shorter.

WHEN THE DAYS ARE NOT DIVISIBLE BY THREE

NOTE.—When the time consists of months and days, and the days are not divisible by three, *reduce the time to days.*

EXAMPLE 5.—What is the interest of $960 for 11 months and 20 days at 6 per cent?

Operation. Months. 11 Days. 20=350 days.

$$\begin{array}{r|l} \cancel{6}\text{ —}36 & \cancel{9}\cancel{6}\cancel{0}\text{—}160 \\ & 350 \\ & 6 \end{array} \qquad \begin{array}{r} 350 \\ 160 \\ \hline \$56.000 \end{array}$$

Now cancel 6 in 36 and the quotient 6 into 960, and we have no divisor left. Hence 160×350 must be the answer.

EXAMPLE 6.—What is the interest of $173 for 8 months and 16 days at 9 per cent?

Operation. Months. 8 Days. 16=256 days.

$$\begin{array}{r|l} \cancel{4}\text{—}\cancel{3}\cancel{6} & 173 \\ & \cancel{9} \\ & \cancel{2}\cancel{5}\cancel{6}\text{—}64 \end{array} \qquad \begin{array}{r} 173 \\ 64 \\ \hline \$11.072 \textit{ Ans.} \end{array}$$

Now cancel 9 in 36 and the quotient 4 into 256, and we have no divisor left. Hence 64×173 must be the answer.

N. B. Let the pupil remember that this is a general and universal method, equally applicable to any per cent. or any required time, and all other rules must be reconcilable to it; and, in fact all other rules are but modifications of this.

EXAMPLE 7.—What is the interest on $1080 for 7 months and 11 days at 7 per cent?

Months. Days.
7 11=221 days.

Operation.

	$\cancel{1}\cancel{0}\cancel{8}\cancel{0}$—30	
$\cancel{3}\cancel{6}$	221	
	7	

221
30
———
6630
7
———
$46,410 *Ans.*

Now cancel 36 in 1080 and we have no divisor left, hence $30 \times 221 \times 7$ must be the answer.

WITH MORE DIFFICULT TIME AND RATE PER CENT.

EXAMPLE 8.—What is the interest of $160 for 19 months and 23 days at $4\frac{1}{2}$ per cent?

Months. Days.
19 23=593 days.

Operation.

	160—20
$\cancel{8}$—$\cancel{3}\cancel{6}$	593
	$\cancel{4\frac{1}{2}}$

593
20
———
$11.860 *Ans.*

Now cancel $4\frac{1}{2}$ in 36 and the quotient 8 into 160 we have no divisor left, hence 20×593 must be the interest.

WHEN THE DAYS ARE DIVISIBLE BY THREE.

RULE.—*Place one-third of the days to the right of the months, and place 12 on the left of the line.*

NOTE.—Dividing the days by 3 reduces them to tenths of months, as 3 days=1 tenth of a month, 6 days=2 tenths, etc.

EXAMPLE 9.—What is the interest of $240 for 1 year 11 months and 12 days at 5 per cent?

Years.	Months.	Days.
1	11	12=23.4 months.

Operation.

$$\begin{array}{r|l} & \cancel{2}\cancel{4}\cancel{0}\text{—}20 \\ \cancel{1}\cancel{2} & 234 \\ & \ \ 5 \end{array}$$

$$\begin{array}{r} 20 \\ 5 \\ \hline 100 \\ 234 \\ \hline \$23.400 \text{ Ans.} \end{array}$$

Now Cancel 12 in 240 and we have no divisor left, hence $20 \times 5 \times 234$ must be the interest.

EXAMPLE 10.—What is the interest of $500 for 2 years 5 months and 24 days at 6 per cent?

Years.	Months.	Days.
2	5	24=29.8 months.

Operation.

$$\begin{array}{r|l} & 500 \\ \cancel{2}\ \cancel{1}\cancel{2} & \cancel{2}\cancel{9}\cancel{8}\text{—}149 \\ & \ \ 6 \end{array}$$

$$\begin{array}{r} 149 \\ 500 \\ \hline \$74.500 \text{ Ans.} \end{array}$$

Now cancel 6 in 12 and the quotient 2 into 298 and we have no divisor left, hence 500×149 equals the interest.

EXAMPLE 11.—What is the interest of $350 for 3 years 7 months and 6 days at 10 per cent?

Years.	Months.	Days.
3	7	6=43.2 months.

Operation.

$$\begin{array}{r|l} & 350 \\ \cancel{12} & \cancel{432}—36 \\ & 10 \end{array} \qquad \begin{array}{r} 350 \\ 36 \\ \hline 12600 \\ 10 \\ \hline \$126.000 \end{array}$$ *Ans.*

Now cancel 12 in 432 and we have no divisor left. Hence 350×36×10 equals the interest.

EXAMPLE 12.—What is the interest of $241 for 13 months and 9 days at 8 per cent?

Months.	Days.
13	9=13. 3 months.

Operation.

$$\begin{array}{r|l} & 241 \\ 3–\cancel{12} & 13.3 \\ & \cancel{8}–2 \end{array} \qquad \begin{array}{r} 241 \\ 133 \\ \hline 32053 \\ 2 \\ \hline 3)64106 \\ \hline \$21.368\tfrac{2}{3} \end{array}$$ *Ans.*

In this example I canceled 8 and 12 by 4, and then multiplied all on the right of the line and di

vided by 3. If I could have divided by 3 before multiplying I would have saved labor, but when the numbers are prime the whole work must be *literally* done.

Closing Remarks.—We have now fully explained the canceling system of computing interest. Any and every problem can be stated by this method, and the beauty and simplicity of the system ranks it high among the most important abbreviations ever discovered by man. As we have before remarked, at 6, 4, 8, 9, 12, 15, and $4\frac{1}{2}$ per cents., every problem in interest can be canceled, besides a great many can be abbreviated at 5, 7, and other per cents.; and after the problem has been stated and we find that we can not cancel, what have we done? We have simply stated the problem in its *simplest* and *easiest* form for working it by *any* other method. Hence we have a decided advantage of *all* notes that will cancel, and if we can not cancel we have stated the problem in its correct and proper form for going through the whole work; but it is only when the principal, time, and rate per cent. are *all* prime, that the WHOLE work must be LITERALLY done. At 6 per cent. we can cancel through, and 6 is the rate *most commonly used*

SHORT PRACTICAL RULES,

DEDUCED FROM THE CANCELING SYSTEM,

For calculating interest at 6 *per cent., either for months, or months and days.*

To find the interest for months at 6 per cent.

RULE.—*Multiply the principal by half the number of months, expressed decimally as a per cent.; that is, for* 12 *months, multiply by* .06; *for* 8 *months, multiply by* .04.

NOTE 1.—It is obvious that if the rate per cent. were 12, it would be 1 per cent. a month; if, therefore, it be 6 per cent., it will be a half per cent. a month; that is, half the months will be the per cent.

NOTE 2.—If any other per cent. is wanted you can proceed as above, and then multiply by the given rate per cent. and divide by 6, and the quotient is the interest.

1. What is the interest of $363 for 8 months?

$$
\begin{array}{r l}
\$368 & \\
.04 & = \text{half the months.} \\
\hline
\$14.72 & = \textit{Ans.}
\end{array}
$$

NOTE 3.—When the months are not even; that is, will not divide by 2, *multiply one half the principal*

by the whole number of months, expressed decimally.

To find the interest of any sum at 6 per cent. per annum for any number of months and days.

RULE.—*Divide the days by* 3 *and place the quotient to the right of the months; one-half of the number thus formed multiplied by the principal, or one-half of the principal multiplied by this number, will give the interest—pointing off three decimal places when the principal is* $.

2. What is the interest of $76 for 1 year, 6 months, and 12 days, at 6 per cent?

Years.	Months.	Days.	
1	6	12	=18.4 months—half 9.2.

$76	*Or,*	184
9.2		38=half prin.
$6.992 *Ans.*		$6.992 *Ans.*

NOTE.—Dividing the days by 3 reduces them to the tenth of months.

To find the the interest of any sum at 6 per cent. per annum for any number of days.

RULE.—*Divide the principal by* 6 *and multiply the quotient by the number of days; or divide the days by* 6 *and multiply the quotient by the principal, pointing off three decimal places when the principal is* $.

NOTE.—Always divide 6 into the number that

will divide without a remainder; if neither one will divide, multiply the principal and days together and divide the result by 6.

3. What is the interest of $240 for 18 days at 6 per cent?

$18 \div 6 = 3$	$240 \div 6 = 40$
$240 *Or,*	$40 = $\frac{1}{6}$ of prin.
3 = $\frac{1}{6}$ of the days.	18
$0.720 *Ans.*	$0.720 *Ans.*

4. What is the interest of $1800 for 72 days at 6 per cent.

$1800 *Or,*	$300 = $\frac{1}{6}$ of prin.
12 = $\frac{1}{6}$ of the days.	72
$21.600 *Ans.*	$21.600 *Ans.*

Useful Suggestions to the Accountant in Computing Interest at 6 per cent.

If the principal is divisible by 6, *always reduce the time to days;* then multiply the number of days by one-sixth of the principal.

EXAMPLE.

5. Find the interest of $240 for 1 year, 5 months, and 17 days, at 6 per cent.

6)240	1yr, 5mos., 17da. = 527 days.
	Multiplied by 40
$\frac{1}{6}$ of prin. = 40	
	$21.080 *Ans.*

If the days are only divisible by 3, multiply one-third of the principal by one-half of the days.

6. What is the interest of $210 for 80 days at 6 per cent.?

$70=⅓ of the principal.
40=½ of the days.

$2.800 *Ans.*

When the Rate of Interest is 4 per cent

RULE.—*Multiply the principal by one-third the number of months, or by one-ninth the number of days, and the product is the interest.*

NOTE.—This principle is also deduced from the canceling method of computing interest; the student can readily see that 4 is ⅓ of 12 and $\frac{1}{9}$ of 36.

When the Rate of Interest is 9 per cent.

RULE.—*Multiply the principal by three-fourths the number of months, or one-fourth the number of days, or vice versa.*

BANKERS' METHOD

OF

COMPUTING INTEREST,

AT 6 PER CENT. FOR ANY NUMBER OF DAYS.

RULE.—*Draw a perpendicular line, cutting off the two right hand figures of the \$, and you have the interest of the sum for* 60 *days at* 6 *per cent.*

NOTE.—The figures on the left of the line are \$, and those on the right are decimals of \$.

EXAMPLE 1.—What is the interest of \$423 60 days at 6 per cent.?

\$423=the principal.

\$4 | 23 cts.=interest for 60 days.

NOTE.—When the time is more or less than 60 days, first get the interest for 60 days, and from that to the time required.

EXAMPLE 2.—What is the interest of \$124 for 15 days at 6 per cent.?

Days. Days.
15=¼ of 60

\$124=principal.
4)1 | 24 cts.=interest for 60 days.

| 31 cts.=interest for 15 days.

EXAMPLE 3.—What is the interest of $123.40 for 90 days at 6 per cent.?

		Days. Days. Days.
		90=60+30
$123.40=principal.		
2)1	2340=interest for 60 days.	
	6170=interest for 30 days.	
Ans. $1	851=interest for 90 days.	

EXAMPLE 4.—What is the interest of $324 for 75 days at 6 per cent.?

		Days. Days. Days.
$324=principal.		75=60+15
4)3	24 cts. interest for 60 days.	
	81 cts. interest for 15 days.	
Ans. $4	05 cts. interest for 75 days.	

REMARKS.—This system of Computing Interest is very easy and simple, especially when the days are aliquot parts of 60, and one simple division will suffice. It is used extensively by a large majority of our most prominent bankers; and, indeed, is taught by most all Commercial Colleges as the shortest system of computing interest.

Method of Calculating at Different Per Cents.

This principle is not confined alone to 6 per cent. as many suppose who teach and use it. It is their custom *first* to find the interest at 6 per cent., and from that to other per cents. But it is equally applicable for *all* per cents., from 1 to 15 inclusive.

The following table shows the different per cents., with the time that a given number of $ will amount to the same number of cents when placed at interest.

RULE.—*Draw a perpendicular line, cutting off the two right hand figures of $, and you have the interest at the following percents.*

Interest at 4 per cent. for 90 days.
Interest at 5 per cent. for 72 days.
Interest at 6 per cent. for 60 days.
Interest at 7 per cent. for 52 days.
Interest at 8 per cent. for 45 days.
Interest at 9 per cent. for 40 days.
Interest at 10 per cent. for 36 days.
Interest at 12 per cent. for 30 days.
Interest at 7-30 per cent. for 50 days.
Interest at 5-20 per cent. for 70 days.
Interest at 10-40 per cent. for 35 days.
Interest at $7\frac{1}{2}$ per cent. for 48 days.
Interest at $4\frac{1}{2}$ per cent. for 80 days.

NOTE.—The figures on the left of the perpendicular line are dollars, and on the right decimals of $. If the $ are less than 10 prefix a 0.

EXAMPLE 1.—What is the interest of $120 for 15 days at 4 per cent.?

$120=principal. Days. 15 = $\frac{1}{6}$ of Days. 90.

6)1 | 20 cts.=int. for 90 days.
  | 20 cts.=int. for 15 days.

EXAMPLE 2.—What is the interest of $132 for 13 days at 7 per cent.?

		Days. Days.
$132=principal.		13=¼ of 52.
4)1	32 cts.=int. for 52 days.	
	33 cts.=int. for 13 days.	

EXAMPLE 3.—What is the interest of $520 for 9 days at 8 per cent.?

		Days. Days.
$520=principal.		9=⅕ of 45.
5)5	20 cts.=int. for 45 days.	
$1	04 cts.=int. for 9 days.	

EXAMPLE 4.—What is the interest of $462 for for 64 days at 7½ per cent.?

		Days. Days. Days.
$462=principal.		64=48+16.
3)4	62 cts. =int. for 48 days.	
1	54 cts. =int. for 16 days.	
$6	16 cts.=int. for 64 days.	

REMARK.—We have now illustrated several examples by the different per cents.; and if the student will study carefully the solution to the above examples, he will in a short time be very rapid in this mode of computing interest.

NOTE.—The preceding mode of computing interest is derived and deduced from the canceling system; as the ingenious student will readily see. It is a short and easy way of finding interest for days when the days are even or aliquot parts; but when they are not multiples, and three or four di

visions are ncessary, the canceling system is much more simple and easy. We will here illustrate an example to show the difference: Required the interest of $420 for 49 days at 6 per cent.

Bankers' method.

2)4	20	cts.=int. for 60 days.
--	—	
2)2	10	cts.=int. for 30 days.
5)1	05	cts.=int. for 15 days.
3)	21	cts.=int. for 3 days.
	7	cts.=int. for 1 day.
$3	43	cts.=int. for 49 days.

Canceling meth.

	~~420~~—70
~~6~~—~~36~~	~~6~~
	49
	70

$3.430 *Ans.*

The canceling method is much more brief; we simply cancel 6 in 36, and the quotient 6 into 420; there is no divisor left; hence 70×49 gives the interest at *once.*

If the time had been 15 or 20 days, the Bankers' Method would have been equally as short, because 15 and 20 are aliquot parts of 60. The superiority the canceling system has above all others is this: it takes advantage of the *principal* as well as the *time.*

For the benefit of the student, and for the convenience of business men, we will investigate this system to its full extent and explain how to take advantage of the *principal* when no advantage can be taken of the *days.* This is one of the most important characteristics of interest, and very often saves much labor. *It should be used when the days are not even or aliquot parts.*

The following table shows the different sums of money (at the different per cents.) that bear 1 cent interest a day; hence the time in days is always the interest in cents; therefore, to find the interest on any of the following notes at the per cent. attached to it in the table, we have the following rule:

RULE.—*Draw a perpendicular line, cutting off the two right hand figures of the days for cents, and you have the interest for the given time.*

Interest of \$90 at 4 per cent. for 1 day is 1 cent.
Interest of \$72 at 5 per cent. for 1 day is 1 cent.
Interest of \$60 at 6 per cent. for 1 day is 1 cent.
Interest of \$52 at 7 per cent. for 1 day is 1 cent.
Interest of \$45 at 8 per cent. for 1 day is 1 cent.
Interest of \$40 at 9 per cent. for 1 day is 1 cent.
Interest of \$36 at 10 per cent. for 1 day is 1 cent
Interest of \$30 at 12 per cent. for 1 day is 1 cent.
Interest of \$50 at 7.30 per ct. for 1 day is 1 ct.
Interest of \$70 at 5.20 per ct. for 1 day is 1 ct.
Interest of \$35 at 10.40 per ct. for 1 day is 1 ct.
Interest of \$48 at 7½ per. cent. for 1 day is 1 cent.
Interest of \$80 at 4½ per cent. for 1 day is 1 cent.
Interest of \$24 at 15 per ct. for 1 day is 1 cent.

NOTE.—The 7.30 Government Bonds are calculated on the base of 365 days to the year, and the 5.20's and 10.40's on the base of 364 days to the year

NOTE.—This table should be committed to memory, as it is very useful when the days are not even or aliquot parts. If the days are less than 10 prefix a 0 before drawing the line.

EXAMPLE 1.—Required the interest of $60 for 117 days at 6 per cent.

117=the days.
$1 | 17 cts. *Ans.*

Here we cut off the two right hand figures for cents.

The student should bear in mind that the interest on $60 for 117 days is just the same as the interest on $117 for 60 days.

By looking at the table we see that the interest for $60 at 6 per cent. is 1 cent a day; hence the time in days is the answer in cents. If this note was $120, instead of $60, we would first find the interest for $60, and then double it; if it was $180, we would multiply by 3, etc.

EXAMPLE 2.—Required the interest of $45 for 219 days at 8 per cent.

219=the days.
$2 | 19 cts. *Ans.*

Here we cut off the two right hand figures for cents.

The student should bear in mind that the interest on $45 for 219 days is just the same as the interest on $219 for 45 days.

By looking at the table we see that the interest on $45 at 8 per cent. is 1 cent a day; hence the time in days is the answer in cents. If this note

was \$22.50, instead of \$45, we would first get the interest for \$45, and then divide by 2; if it was \$75, we would add on $\frac{2}{3}$; if \$60, add on $\frac{1}{2}$, etc.

EXAMPLE 3.—Required the interest of \$48 for 115 days at 9 per cent.

115=the days. \$48=\$40+\$8.

5)\$1	15 cts.=the int. of \$40 for 115 days.
	23 cts.=the int. of \$8 for 115 days.
Ans. \$1	38 cts.=the int. of \$48 for 115 days.

Here we first find the interest of \$40, because the days is the interest in cents; then we divide by 5 to find the interest for \$8; then by adding both we find the interest for \$48, as required.

EXAMPLE. 4—Required the interest of \$260 for 104 days at 7 per cent.

\$52×5=\$260.

104=the days.

\$1	04 cts=the int. of \$52 for 104 days.
Ans. \$5	20 cts. Multiply by 5.

Here we first find the interest of \$52, because the days is the interest in cents; then we multiply by 5 to get it for \$260. We could have worked this note by the Bankers' Method, just as well, by cutting off two figures in the principal, making \$2.60 cts. the interest for 52 days, and then multiply by 2 to get it for 104 days. The student must remember that the interest of \$260 for 104 days is just the same as the interest of \$104 for 260 days.

Problems Solved by Both Methods.

We will now solve some examples by both methods, to further illustrate this system, and for the purpose of teaching the pupil how to use his judgment. He will then have learned a rule *more valuable than all others.*

EXAMPLE 5.—What is the interest $180 for 75 days at 6 per cent.?

Operation by taking advantage of the $.

75=the days. $60×3=$180.

$0	75 cts.=the int. of $60 for 75 days.	
	3	Multiply by 3.
Ans. $2	25 cts.=the int. of $180 for 75 days.	

Operation by the Bankers' Method.

$180=the principal. 60da.+15da.=75da.

4)$1	80 cts.=the int. for 60 days.
	45 cts.=the int. for 15 days.
Ans. $2	25 cts.=the int. for 75 days.

By the first method we multiplied by 3, because 3×$60=$180; by the second method we added on $\frac{1}{4}$, because 60da.+$\frac{60}{4}$da.=75da.

N. B.—When advantage can be taken of both time and principal, if the student wishes to prove his work, he can first work it by the Bankers' Method, and then by taking advantage of the principal, or *vice versa.* And as the two operations are entirely different, if the same result is obtained by each, he may fairly conclude that the work is correct.

LIGHTNING METHOD

OF

COMPUTING INTEREST

On all notes that bear $12 *per annum, or any aliquot part or multiple of* $12.

If a note bears $12 per annum, it will certainly bear $1 per month; hence the time in months would be the interest in $; and the decimal parts of a month would be the interest in decimal parts of a $; therefore when the note bears $12 per annum we have the following rule:

Rule.—*Reduce the years to months, add in the given months, and place one-third of the days to the right of this number, and you have the interest in dimes.*

Example 1.—Required the interest of $200 for 3 years, 7 months, and 12 days, at 6 per cent.

200 $\frac{1}{3}$ of 12 days=4.
6

Yr. Mo. Da.
$12.00=int. for 1 yr. 3 7 12=43.4mo.
Hence 43.4 dimes, or $43.40cts., *Ans.*

We see by inspection that this note bears $12 interest a year; hence the time reduced to months,

with one-third of the days to the right, is the interest in dimes. If this note bore $6 a year, instead of $12, we would take one-half of the above interest; if it bore $18, instead of $12, we would add one-half; if it bore $24, instead of $12, we would multiply by 2, etc.

EXAMPLE 2.—Required the interest of $150 for 2 years, 5 months, and 13 days, at 8 per cent.

150	$\frac{1}{3}$ of 13 days=$4\frac{1}{3}$.
8	
	Yr. Mo. Da.
$12.00=int. for 1 yr.	2 5 13=$29.4\frac{1}{3}$mos.

Hence $$29.4\frac{1}{3}$ dimes, or $$29.43\frac{1}{3}$ cts., *Ans.*

We see by inspection that this note bears $12 interest a year; hence the time reduced to months, with one-third of the days placed to the right, gives the interest at once.

EXAMPLE 3.—Required the interest of $160 for 11 years, 11 months, and 11 days, at $7\frac{1}{2}$ per cent.

160	$\frac{1}{3}$ of 11 days=$3\frac{2}{3}$.
$7\frac{1}{2}$	
	Yr. Mo. Da.
$12.00=int. for 1 yr.	11 11 11=$143.3\frac{2}{3}$mos.

Hence $$143.3\frac{2}{3}$ dimes, or $$143.36\frac{2}{3}$ cts., *Ans.*

When the Interest is more or less than $12 *a Year.*

RULE.—*First find the interest for the given time on the base of* $12 *interest a year; then, if the interest on the note is only* $6 *a year, divide by* 2; *if*

$24 a year, multiply by 2; if $18 a year, add on one-half, etc.

EXAMPLE 1.—What is the interest of $300 for 4 years, 7 months, and 18 days, at 6 per cent.

$\frac{1}{3}$ of 18 days=6.

300	4yr. 7mo. 18da.=55.6mo.
6	
$18.00=int. for 1 year.	2)55.6, int. at $12 a year.
$18=1½ times $12.	278
	$83.4 *Ans.*

If the interest was $12 a year, $55.60 would be the answer; because 55.6 is the time reduced to months; but it bears $18 a year, or 1½ times 12; hence 1½ times 55.6 gives the interest at once.

EXAMPLE 2.—Required the interest of $150 for 3 years, 9 months, and 27 days, at 4 per cent.

$\frac{1}{3}$ of 27 days=9.

150	3yr. 9mo. 27da.=45.9mo.
4	2)45.9, int. at $12 a year.
$6.00=int. for 1 year.	$22.95 *Ans.*
$6=½ times $12.	

If the interest was $12 a year, $45.90 would be the answer; because 245.9 is the time reduced to months; out it bears $6 a year, or ½ times 12; hence ½ times 45.9 g'ves the interest at once.

MERCHANTS' METHOD

OF

COMPUTING INTEREST.

FOR YEARS, MONTHS, AND DAYS.

THE computation of simple interest, where the time consists of years, months, and days, is quite difficult. Taking the aliquot parts for the different portions of time almost invariably involves the calculator in fractions, and, unless he is well versed in vulgar fractions he will not be able to arrive at the correct result. We have three bases by which we compute interest at different rates per cent. and by which we are enabled to entirely avoid the use of fractions. These three bases are each obtained different from the other, and consequently we have three rules for computing interest: one at a base of one per cent., a second at a base of twelve per cent., and a third at a base of thirty-six per cent.

RULE for computing interest at 1 per cent.:

Take one-third of the number of days and annex to the number of months; divide the number thus formed by 12; *annex the quotient thus obtained to the number of years, and multiply the principal by this number; if the principal contains cents, point off five decimal places; if not, point off three deci-*

mal places; this will give the interest at one per cent. For any other rate per cent., multiply the interest at one per cent. by the required rate per cent.

Remark.—This rule applies to all problems in interest where the days are divisible by 3, and this number, annexed to the number of months, divisible by 12.

EXAMPLE.

Required the interest on $112, at 1 per cent., for 3 years, 3 months and 18 days.

SOLUTION.

Take one-third of the number of days, $\frac{1}{3}$ of 18 =6, annex this number to the months given, 36, divide this number by 12, 36÷12=3, annex this number to the year gives, 33, multiply the principal by 33, $112×33=3.69 6, point off three decimal places, and we have the required interest, $3.69 6.

EXAMPLE.

Required the interest on $125 12, at 7 per cent., for 2 years, 8 months and 12 days.

SOLUTION.

Take one-third of the number of days, $\frac{1}{3}$ of 12=4, annex this number to the number of months we have 84, divide this number by 12,

84÷12=7, annex this number to the number of years we have 27, multiply the principal by this number, and point off five decimal places, and you have the interest at one per cent.; multiply this interest by 7, and you have the interest at 7 per cent., the required rate.

$$\begin{array}{r} \$125\ 12 \\ 27 \\ \hline 3.37824 \\ 7 \\ \hline \$23\ .64768 \end{array}$$

EXAMPLE.

Required the interest on \$1,023, at 8 per cent., for 1 year, 9 months and 18 days.

SOLUTION.

Take one-third the number of days and annex to the number of months, $\frac{1}{3}$ of 18=6, we have 96÷12=8, annex this number to the years we have 18, multiply the principal by this number, and point of three decimal places, which gives the interest at 1 per cent.; multiply the interest at one per cent. by 8, and you have the required interest.

$$\begin{array}{r} \$1023 \\ 18 \\ \hline \$18\ .414 \\ 8 \\ \hline \$147\ .312 \end{array}$$

Remark.—This rule will apply to all problems in interest if one-third of the number of the days be taken decimally and annexed to the number of months, and this number, divided by 12, carried out decimally. But this makes the multiplier very large; hence, to avoid this large number in

the multiplier, where the days are divisible by 3, and this number, annexed to the months, is not divisible by 12, we use the following rule, called our base at 12 per cent.:

RULE.—*Reduce the years to months, add in the months, take one-third of the number of days and annex to this number, multiply the principal by the number thus formed; if there are cents in the principal, point off five decimal places; if there are no cents in the principal, point off three decimal places; this gives the interest at 12 per cent. For any other rate per cent., take such part of the base before multiplying as the required rate is a part of 12.*

EXAMPLE.

Required the interest on $123, at 12 per cent., for 2 years, 2 months and six days.

SOLUTION.

Reduce the 2 years to months gives us 24 months, add on the 2 months gives us 26 months, take one-third of the days, $\frac{1}{3}$ of 6=2, annexed to the 26 months gives 262, which constitutes the base; multiply the principal by this base, and you have the interest at 12 per cent.

$123
262
———
$32.226

EXAMPLE.

Required the interest on $144, at 6 per cent., for 4 years, 5 months and 12 days.

SOLUTION.

Reduce the 4 years to months gives 48 months, add in the 5 months gives 53 months, take one-third of the days and annex to the number of months, $\frac{1}{3}$ of 12=4, annex to the 53 months, 534; this number multiplied into the principal would give the interest at 12 per cent. But we want it at 6 per cent. We will now take such part of either principal or base as 6 is a part of 12; 6 is $\frac{1}{2}$ of 12, therefore we will take $\frac{1}{2}$ of 144=72 one-half of the principal, and multiply it by the base, which will give the interest at 6 per cent.

$$\begin{array}{r} 72 \\ 534 \\ \hline \$38.448 \end{array}$$

EXAMPLE.

Required the interest on \$347 25, at 8 per cent., for 2 years, 3 months and 9 days.

SOLUTION.

Reduce the 2 years to months, 24 months, add the 3 months, 27 months, take one-third of the days, $\frac{1}{3}$ of 9=3, annex to the months, 273, the base; this, multiplied into the principal, would give the interest at 12 per cent. But we want the interest at 8 per cent; we will take two-thirds of the base before multiplying: $\frac{2}{3}$ of 273=182; the principal multiplied by this number gives the interest at 8 per cent.

$$\begin{array}{r} \$347\ 25 \\ 182 \\ \hline \$63.19950 \end{array}$$

Remark.—This base is used where the days are divisible by 3, and the number formed by annex-

ing one-third of the days to the months not divisible by 12. We now come to time in which neither days nor months are divisible. Where such time as this occurs, we use a base at 36 per cent.

RULE.—*Reduce the time to days, by multiplying the years by* 12, *adding in the months, if any, and multiplying this number by* 30, *adding in the days, if any; multiply the principal by this number, pointing off* 5 *decimal places, where cents are given in the principal, and* 3 *places where no cents are given. This will give the interest at* 36 *per cent.*

EXAMPLE.

Required the interest on $144, at 36 per cent., for 3 years, 2 months and 2 days.

SOLUTION.

Reduce the time to days gives 1142 days; multiply the principal by this base, and you have the interest at 36 per cent

	$144
	1142
	$164.448

EXAMPLE.

Required the interest on $144, at 9 per cent., for 5 years, 7 months and 5 days.

SOLUTION.

Reduce the time to days gives 2,015 days; if we multiply the principal by this base, we would get the interest at 36 per cent.; but we want it at 9 per cent. We can take such part of either

principal or base as 9 is a part of 36 before multiplying; 9 is $\frac{1}{4}$ of 36; we will take $\frac{1}{4}$ of the principal, it being divisible by 4; $\frac{1}{4}$ of 144=36, which, multiplied into the base, will give the interest at 9 per cent., by pointing off 3 decimal places.

$$\begin{array}{r} 2015 \\ 36 \\ \hline \$72.540 \end{array}$$

EXAMPLE.

Required the interest on $875 15, at 6 per cent., for 5 years, 7 months and 12 days.

SOLUTION.

Reduce the time to days gives 2022 days; 6 is $\frac{1}{6}$ of 36; take one sixth of the base, $\frac{1}{6}$ of 2022=337; multiply the principal by this number, point off 5 decimal places, and you have the interest at 6 per cent., the required rate.

$$\begin{array}{r} \$875\ 15 \\ 337 \\ \hline \$294.92555 \end{array}$$

Remark.—We have now fully explained our method of computing interest at the three different bases. Any and every problem in interest can be solved by one of these three bases. Some problems can be solved easier by one base than another. Where the days are divisible by 3, and their number, annexed to the months, divisible by 12, it is the shortest and best method to use the base at 1 per cent. By using one or the other of these three bases, the student can avoid the use of vulgar fractions. The student must study these three principles carefully, and learn to adopt readily the base best suited to the problem to be solved.

PARTIAL PAYMENTS

ON NOTES, BONDS, AND MORTGAGES.

To compute interest on notes, bonds, and mortgages, on which partial payments have been made, two or three rules are given. The following is called the common rule, and applies to cases where the time is short, and payments made within a year of each other. This rule is sanctioned by custom and *common law;* it is true to the principles of simple interest, and requires no special enactment. The other rules are rules of *law*, made to suit such cases as require (either expressed or implied) annual interest to be paid, and of course apply to no business transactions closed within a year.

RULE.—*Compute the interest of the principal sum for the whole time to the day of settlement, and find the amount. Compute the interest on the several payments, from the time each was paid to the day of settlement; add the several payments and the interest on each together, and call the sum the amount of the payments. Subtract the amount of the payments from the amount of the principal, will leave the sum due.*

EXAMPLES.

1. A gave his note to B for $10,000; at the end of 4 months, A paid $6,000; and at the expiration of another 4 months, he paid an additional sum of $3,000; how much did he owe B at the close of the year?

By the Common Rule.

Principal		$10,000
Interest for the whole time		600
Amount		$10,600
1st payment	$6,000	
Interest, 8 months	240	
2d payment	3,000	
Interest, 4 months	60	
Amount	$9,300	9,300
Due		$1 300

PROBLEMS IN INTEREST.

There are *four* parts or quantities connected with each operation in interest: these are, the *Principal*, *Rate per cent.*, *Time*, *Interest* or *Amount.*

If any *three* of them are given the *other* may be found.

Principal, interest, and time given, to find the rate per cent.

1. At what rate per cent. must $500 be put on interest to gain $120 in 4 years?

Operation.

$500
.01
———
5.00
4
———
20.00)120.00(6 per cent., *Ans.*
120.00
———

By analysis.

The interest of $1 for the given time at 1 per cent. is 4 cents. $500 will be 500 times as much=500×.04 =$20.00. Then if $20 give 1 per cent., $120 will give $\frac{120}{20}$ =6 per cent.

RULE.—*Divide the given interest by the interest of the given sum at 1 per cent. for the given time, and the quotient will be the rate per cent. required*

Principal, interest, and rate per cent. given, to find the time.

2. How long must $500 be on interest at 6 per cent. to gain $120?

Operation

$500
.06
———
30.00)120.00(4 years, *Ans.*
120.00
———

By analysis.

We find the interest of $1.00 at the given rate for 1 year is 6 cents $500, will therefore be 500 times as much=500×.06=$30.00. Now, if it take 1 year to gain $30, it will require $\frac{120}{30}$ to gain $120=4 years, *Ans.*

RULE.—*Divide the given interest by the interest of the principal for* 1 *year, and the quotient is the time.*

Given the amount, time, and rate per cent., to find the *principal.*

RULE.—*Divide the given amount by the amount of* \$1, *at the given rate per cent., for the given time.*

REMARK.—This rule is deduced from the fact that the amount of different principals for the same time and at the same rate per cent., are to each other as those principals.

BANK DISCOUNT.

Bank Discount is the sum paid to a bank for the payment of a note before it becomes due.

The amount named in a note is called the *face* of the note. The *discount* is the interest on the face of the note for 3 days more than the time specified, and is paid in advance. These 3 days are called *days of grace*, as the borrower is not obliged to make payment until their expiration. Hence, to compute bank discount, we have the following

RULE.—*Find the interest on the face of the note for* 3 *days more than the* TIME *specified; this will be the discount. From the face of the note deduct the discount, and the remainder will be the* PRESENT VALUE *of the note.*

DISCOUNT, OR COUNTING BACK.

The object of discount is to show us what allowance should be made when any sum of money is paid before it becomes due.

The *present worth* of any sum is the principal that must be put at interest to amount to that sum in the given time. That is, \$100 is the *present worth* of \$106 due one year hence; because \$100 at 6 per cent. will amount to \$106; and \$6 is the *discount.*

1. What is the present worth of \$12.72 due one year hence?

First method.

```
   $12.72
      100
   ------
106)1272.00($12 Ans.
    106
    ---
     212
     212
     ---
```

Second method.

```
         $
1.06)12.72($12 Ans.
     10.6
     ----
      2.12
      2.12
      ----
```

As \$100 will amount to \$106 in one year at 6 per cent., it is evident that if $\frac{100}{106}$ of any sum be taken, it will be its present worth for one year, and that $\frac{6}{106}$ will be the discount. And as \$1 is the present worth of \$1.06 due one year hence, it is evident that the present worth of \$12.72 must be equal to the number of times \$12.72 will contain \$1.06.

RULE.—*Divide the given sum by the amount of* $1 *for the given rate and time, and the quotient will be the present worth. If the present worth be subtracted from the given sum, the remainder will be the discount.*

EQUATION OF PAYMENTS.

EQUATION OF PAYMENTS is the process of finding the equalized or average time for the payment of several sums due at different times, without loss to either party.

To find the average or mean time of payment, when the several sums have the same date.

RULE.—*Multiply each payment by the time that must elapse before it becomes due; then divide the sum of these products by the sum of the payments, and the quotient will be the averaged time required.*

NOTE.—When a payment is to be made down, it has no product, but it must be added with the other payments in finding the average time.

EXAMPLE 1.—I purchased goods to the amount of $1200; $300 of which I am to pay in 4 months $400 in 5 months, and $500 in 8 months. How long a credit ought I to receive, if I pay the whole sum at once? *Ans.* 6 months.

Mo.	Mo.	
4×300	=1200	A credit on $300 for 4 months is the same as the credit on $1 for 1200 months.
5×400	=2000	A credit on $400 for 5 months is the same as the credit on $1 for 2000 months.
8×500	=4000	A credit on $500 for 8 months is the same as the credit on $1 for 4000 months.

1200) 7200 (6 mo.
7200

Therefore, I should have the same credit as a credit on $1 for 7200 months, and on $1200, the whole sum, one-twelfth hundredth part of 7200 months, which is 6 months.

This rule is the one usually adopted by merchants, although not strictly correct, still, it is sufficiently accurate for all practical purposes.

To find the average or mean time of payment, when the several sums have different dates.

EXAMPLE 1.—Purchased of James Brown, at sundry times, and on various terms of credit, as by the statement annexed. When is the *medium* time of payment?

Jan. 1,	a bill	am'ting to	$360, on 3 months' credit.
Jan. 15,	do.	do.	186, on 4 months' credit.
March 1,	do.	do.	450, on 4 months' credit.
May 15,	do.	do.	300, on 3 months' credit
June 20,	do.	do.	500, on 5 months' credit.

Ans. July 24th, or in 115 da.

Due April 1,	$360	
May 15,	186×44=	8184
July 1,	450×91=	40950
Aug. 15,	300×136=	40800
Nov. 20,	500×233=	116500

1796 ÷ into)206434(114$\frac{845}{898}$ days.

We first find the time when each of the bills will become due. Then, since it will shorten the operation and bring the same result, *we take the time when the first bill becomes due*, instead of its *date*, for the *period* from which to compute the average time. Now, since April 1 is the period from which the average time is computed, no time will be reckoned on the first bill, but the time for the payment of the second bill extends 44 days beyond April 1, and we multiply it by 44.

Proceeding in the same manner with the remaining bills, we find the average time of payment to be 114 days and a fraction, from April 1, or on the 24th of July.

RULE.—*Find the time when each of the sums becomes due, and multiply each sum by the number of days from the time of the earliest payment to the payment of each sum respectively. Then proceed as in the last rule, and the quotient will be the average time required, in days, from the earliest payment.*

NOTE.—Nearly the same result may be obtained by reckoning the time in months.

In mercantile transactions it is customary to give a credit of from 3 to 9 months, on bills of sale. Merchants in settling such accounts, as consist of various items of debit and credit for different times, generally employ the following:

RULE.—*Place on the debtor or credit side, such a sum, (which may be called* MERCHANDISE BALANCE,) *as will balance the account.*

Multiply the number of dollars in each entry by the number of days from the time the entry was made to the time of settlement; and the Merchandise balance by the number of days for which credit was given. Then multiply the difference between the sum of the debit, and the sum of the credit products, by the interest of $1 *for* 1 *day; this product will be the* INTEREST BALANCE.

When the sum of the debit products exceed the sum of the credit products, the interest balance is in favor of the debit side; but when the sum of the credit products exceed the sum of the debit products, it is in favor of the credit side. Now to the merchandise balance add the interest balance, or subtract it, as the case may require, and you obtain the CASH BALANCE.

A has with B the following account:

1849.		*Dr.*	1849.		*Cr.*
Jan. 2.	To merchandise,	$200	Feb. 20.	By merchandise,	$100
April 20.	" "	400	May. 10.	" "	300

If interest is estimated at 7 per cent., and a credit of 60 days is allowed on the different sums, what is the cash balance August 20, 1849?

Ans. 206.54.

EXPLANATION.—Without interest the cash balance would be $200.

If no credit had been given, the debits should be increased by the interest of $200 for 230 days, at 7 per cent.; and the interest of $400 for 122 days, at 7 per cent. The credits should be increased by the interest of $100 for 181 days, at 7 per cent., and the interest of $300 for 102 days, at 7 per cent.

Since a credit of 60 days is given on all sums, it is evident by the above calculation, that we should increase the debits by the interest of the sum of the debits, $600, for 60 days more than justice requires. Also, that we should increase the credits by the interest of the sum of the credits, $400, for 60 days more than we should do.

Now, instead of deducting these items of interest from the *amount* of debit and credit interests, it is plain that it will be more convenient and equally just, to diminish the debit interest of the *merchandise balance* for 60 days, which can be most readily accomplished by adding the interest on the merchandise balance for 60 days, to the credit items of interest.

From which we discover that the *interest balance* is equal to the difference between the sum of the debit interests, and the sum of the credit interests increased by the interest of the merchandise balance for the time for which credit was given.

Operation.

DEBITS.		CREDITS.
\$ Days.		\$ Days.
$200 \times 230 = 46000$		$100 \times 181 = 18100$
$400 \times 122 = 48800$		$300 \times 102 = 30600$
	Balance,	$200 \times 60 = 12000$
94800		
60700		60700

$$\frac{0.07}{365} \times 34100 = \$6.54 \text{ \textit{Interest balance, yearly.}}$$

Therefore, the foregoing account becomes balanced as follows:

1849.		Dr.	1849.		Cr.
Jan. 2.	To Merchandise,	\$200.00	Feb. 20.	By Merchandise,	\$100.00
April 20.	" "	400.00	May. 10.	" "	300.00
Aug. 20.	" balance of int.	6.54	Aug. 20.	" balance,	206.54
		\$606.54			\$606.54
Aug. 20.	" Cash balance,	\$206.54			

NOTE.—It is customary in practice, when the number of cents in any of the entries, are less than 50, to omit them, and to add \$1 when they are 50, or more.

SQUARE AND CUBE ROOTS.

To work the square and cube roots with ease and facility, the pupil must be familiar with the following properties of numbers:

Their importance can not be exaggerated if we wish to insure skill or even sound information on this subject.

I. A square number, multiplied by a square number, the product will be a square number.

II. A square number, divided by a square number, the quotient is a square.

III. A cube number, multiplied by a cube, the product is a cube.

IV. A cube number, divided by a cube, the quotient will be a cube.

V. If the square root of a number is a composite number, the square itself *may be divided into integer square factors*; but if the root is a *prime number*, the square can not be separated into square factors *without fractions.*

VI. If the unit figure of a square number is 5, we may multiply by the square number 4, and we shall have another square, whose *unit* period will be ciphers.

VII. If the unit figure of a cube is 5, we may multiply by the cube number 8, and produce another cube, whose unit period will be ciphers.

N. B. If a supposed cube, whose unit figure is 5, be multiplied by 8, and the product does not give three ciphers on the right, the number *is not a cube.*

We present the following table, for the pupil to compare the natural numbers with the *unit figure* of their *squares* and *cubes*, that he may be able to *extract roots by inspection.*

Numbers...........	1	2	3	4	5	6	7	8	9	10
Squares	1	4	9	16	25	36	49	64	81	100
Cubes	1	8	27	64	125	216	343	512	729	1000

EXERCISES FOR PRACTICE.

1. What is the square root of 625? *Ans.* 25

If the root is an *integer number*, we may know, by the inspection of the table, that it must be 25, as the greatest square in 6 is 2, and 5 is the only figure whose square is 5 in its unit place.

Again, take	625	
Multiply by	4	4 being a square.
	2500	

The square root of this product is obviously 50; but this must be divided by 2, the square root of 4, which gives 25, the root.

2. What is the square root of 6561? *Ans.* 81.

As the *unit figure*, in this example, is 1, and in

the line of squares in the table, we find 1 only at 1 and 81, we will, therefore, divide 6561 by 81, and we find the quotient 81; 81 is, therefore, the square root.

3. What is the square root of 106729? *Ans.* 327.

As the unit figure, in this example, is 9, if the number is a square, it must divide by either 9, or 49. After dividing by 9 we have 11881 for the other factor, a prime number, therefore its root is a prime number=109. 109, multiplied by 3, the root of 9, gives 327 for the answer.

4. What is the root of 451584? *Ans.* 672.

As the unit figure is 4, and in the line of squares we find 4 only at 4 and 64, the above number, *if a square*, must divide by 4, or 64, or by both.

We will divide it by 4, and we have the factors 4 and 112896. This last factor *closes in* 6; therefore, by looking at the table, we see it must divide by 16, or 36, etc.

We divide by 36, and we have the factors 36 and 3136; divide this last by 16, and we have 16 and 196; divide this last fraction by 4, and we have 4 and 49.

Take now our divisors, and last factor, 49, and we have for the original number the product of $4\times36\times16\times4\times49$; the roots of which are $2\times6\times4\times2\times7$, the products of which are 672, the answer

5. Extract the square root of 2025. *Ans.* 45.

1st. Divide by the square number 25, and we find the two factors, 25×81, as equivalent to the given number. Roots of these factors, $5\times9=45$, the answer.

Again, multiply by the square number 4, when a number ends in 25, and we have 8100, root 90, half of which, because we multiplied by 4, the square of 2, is 45, the answer.

Problems on the Right-angled Triangle.

1. The top of a castle is 45 yards high, and is surrounded with a ditch, 60 yards wide; required the length of a ladder that will reach from the outside of the ditch to the top of the castle.

Ans. 75 yards.

This is almost invariably done by squaring 45 and 60, adding them together, and extracting the square root; but so much labor *is never necessary when the numbers have a common divisor*, or when the side sought is expressed by a *composite* number.

Take 45 and 60; both may be divided 15, and they will be reduced to 3 and 4. Square these, $9+16=25$. The square root of 25 is 5, which, multiplied by 15, gives 75, the answer.

Abbreviations in Cube Root

1. What is the cube root of 91125? *Ans.* 45.

$$\begin{array}{lr} \text{Multiply by} & 8 \\ \hline & 729000 \end{array}$$

Now, 729 being the cube of 9, the root of 729000 is 90; divide this by 2, the cube root of 8, and we have 45, the answer.

When it is requisite to multiply several numbers together and extract the cube root of their product, try to change them into *cube factors* and extract the root *before* multiplication.

EXAMPLES.

1. What is the side of a cubical mound equal to one 288 feet long, 216 feet broad, and 48 feet high?

The common way of doing this, is to multiply these numbers together and extract the root—a lengthy operation. But observe that 216 is a cube number, and $288=2\times12\times12$, and $48=4\times12$; therefore the whole product is $216\times8\times12\times12\times12$. Now, the cube root of 216 is 6, of 8 is 2, and of 12^3 is 12, and the product of $6\times2\times12=144$, the answer.

2. Required the cube root of the product of 448×392 the short way. *Ans.* 56.

We can extract the root of cube numbers by inspection when they do not contain more than two periods.

RULE.—*As there will be two figures in the root, the first may easily be found mentally, or by the Table of Powers; and if the unit figure of the power be* 1, *the unit figure in the root will be* 1; *and if it be* 8, *the root will be two*; *and if* 7, *it will be* 3; *and if the unit of the power be* 6, *the unit of the root will be* 6; *and if* 5, *it will be* 5; *if* 3, *it will be* 7; *if* 2, *it will be* 8; *and if the unit of the power be* 9, *the unit of the root will be* 9. *This will appear evident by inspecting the Table of Powers.*

EXAMPLES.

Find the cube root of 195112. This number consists of two periods. Compare the superior period with the cubes in the table, and we find that 195 lies between 125 and 216. The cube root of the tens, then, must be 5. The unit figure of the given cube is 2; and no cube in the table has 2 for its unit figure, except 512, whose root is 8; therefore 58 is the root required.

What is the cube root of 97336? *Ans* 46.

EXPLANATION.—By examining the left hand period, we find the root of 97 is 4, and the cube of 4 is 64. The root can not be 5, because the cube of 5 is 125. The unit figure of the given cube is 6; and no cube in the table has 6 for its unit figure, except 216, whose root is 6; the answer, therefore, is 46.

The number 912673 is a cube; what is its root? *Ans.* 97.

Observe, the root of the superior period must

be 9, and the root of the unit period must be some number which will give 3 for its unit figure *when cubed*; and 7 is the only figure that will answer.

The following numbers are cubes; required their roots.

1. What is the cube root of 59319? *Ans.* 39
2. What is the cube root of 79507? *Ans.* 43.
3. What is the cube root of 117649? *Ans.* 49.
4. What is the cube root of 110592? *Ans.* 48.
5. What is the cube root of 357911? *Ans.* 71.
6. What is the cube root of 389017? *Ans.* 73.
7. What is the cube root of 571787? *Ans.* 83.

When a cube has more than two periods, it can generally be reduced to two by dividing by some one or more of the cube numbers, unless the root is a *prime* number.

The number 4741632 is a cube; required its root. Here we observe that the unit figure is 2; the unit figure of the root must therefore be the root of 512, as that is the only cube of the 9 digits whose unit figure is 2. The cube root of 512 is 8; therefore 8 is the unit figure in the root, and the root is an *even* number, and can be divided by 2; and of course the cube itself can be divided by 8, the cube of 2.

$$8)\underline{4741632}$$
$$592704$$

Now, as the first number was a cube, and being

divided by a cube, the number 592704 must be a cube, and by inspection, as previously explained, its root must be 84, which, multiplied by 2, gives 168, the root required.

The number 13312053 is a cube; what is its root? *Ans.* 237.

As there are three periods, there must be three figures, units, tens, and hundreds, in the root; the hundreds must be 2, the units must be 7. Let us then divide the 2d figure, or the tens, *in the usual way*, and we have 237 for the root.

Again, divide 13312053 by 27, and we have 493039 for another factor. The root of this last number must be 79, which, multiplied by 3, the cube root of 27, gives 237, as before.

The number 18609625 is a cube; what is its root?

As this cube ends with 5, we will multiply it by 8:

$$\begin{array}{r} 18609625 \\ 8 \\ \hline 148877000 \end{array}$$

As the first is a cube, this product must be a cube; and as far as labor is concerned, it is the same as reduced to two periods, and the root, we perceive at once, must be 530, which, divided by 2, gives 265 for the root required.

N. B.—If a number, whose unit figure is 5, be multiplied by 8, and *does not result in three ciphers* on the right, the number is not a cube.

To find the Approximate Cube Root of Surds.

RULE.—*Take the nearest rational cube to the given number, and call it the assumed cube; or assume a root to the given number and cube it. Double the assumed cube and add the number to it; also double the number and add the assumed cube to it. Take the difference of these sums, then say, As double of the assumed cube, added to the number, is to this difference, so is the assumed root to a correction.*

This correction, added to or subtracted from the assumed root, *as the case may require*, will give the cube root very nearly.

By repeating the operation with the root last found as an assumed root, we may obtain results to any degree of exactness; one operation, however, is generally sufficient.

EXAMPLES.

1. Required the cube root of 66.

The cube root of 64 is 4. Now it is manifest that the cube root of 66 is a little more than 4, and by taking a similar proportion to the preceding, we have

$$\begin{array}{rr} 64\times2=128 & 2\times66=132 \\ 66 & 64 \\ \hline 194 & 196 \end{array}$$

194 : 196 : : 4 : to root of 66.

Or, 194 : 2 : : 4 : to a correction

```
194)8.0000(0.04124
    7 76
    ----
     240
     194
     ---
      460
      388
      ---
       720
```

Therefore the cube root of 66 is 4.04124.

2. Required the cube root of 123.

Suppose it 5; cube it, and we have 125.

Now we perceive that the cube of 5 being greater than 123, the *correction* for 5 must be *subtracted.*

```
    2×125=250   246
Add       123   125
          ---   ---
As        373 : 371 : : 5 : root of 123.
```

Or, 373 : 2 : : 5 : correction for 5

```
373)10.0000(0.02681
     7 46
     ----
     2 540              From 5.00000
     2 238              Take 0.02681
     -----                   -------
      3020              Ans. 4.97319
      2984
      ----
       360.
```

3. What is the cube root of 28? *Ans.* 3,03658+
4. What is the cube root of 26? *Ans.* 2,96249+
5. What is the cube root of 214? *Ans.* 5,98142+
6. What is the cube root of 346? *Ans.* 9,02034+

The above being very near integral cubes—that is, 28 and 26 are both near the cube number 27, 214 is near 216, etc. All numbers very near cube numbers are *easy of solution.*

We now give other examples, more distant from integral cubes, to show that the labor must be more lengthy and tedious, though the operation is the same.

1. What is the cube root of 3214? *Ans.* 14,75758.

Suppose the root is 15—its cube is 3375, which, being greater than 3214, shows that 15 is too great; the correction will therefore be subtractive.

By the rule, 9964 : 161 : : 15. 0,243, the correction.

Assumed root...........................	15,0000
Less..	2423
Root nearly..............................	14,7577

Now assume 14,7 for the root, and go over the operation again, and you will have the true root to 8 or 10 places of decimals.

N. B.—Roots of component powers may be obtained more readily thus:

For the 4th root, take the square root of the square root.

APPLICATION OF THE CUBE ROOT.

PRINCIPLES ASSUMED.

Spheres are to each other as the cubes of their diameter.

Cubes, and all solids whose corresponding parts are similar and proportional to each other, are to each other as the cubes of their diameters, or of their homologous sides.

1. If a ball, 3 inches in diameter, weigh 4 pounds, what will be the weight of a ball that is 6 inches in diameter? *Ans.* 32lbs.

2. If a globe of gold, 1 inch in diameter, be worth $120, what is the value of a globe, $3\frac{1}{2}$ inches in diameter? *Ans.* $5145.

Questions Solved by the Rule of Three, Direct or Inverse.

There is a cistern which has a stream of water running into it; it has 10 cocks; all running together will empty it in $2\frac{1}{2}$ hours; 6 will empty it in $5\frac{1}{2}$ hours; how long will it take 3 to empty it? *Ans.* 55 hours.

NOTE.—The 6 cocks will discharge in $4\frac{1}{6}$ hours what the 10 cocks will in $2\frac{1}{2}$ hours; therefore it would take the 6 cocks $1\frac{1}{3}$ to discharge what would run into the cistern in 3 hours: therefore it would take the 6 cocks 1 111 to discharge what would run

in $2\frac{1}{2}$ hours; consequently, $2\frac{2}{3}$ cocks to discharge the water as fast as it run in.

There is a stick of timber, 12 feet long, to be carried by 3 men: one carries at the end, the other two carry by a lever; how far must the lever be placed from the other end that each may carry equally? *Ans.* 3 feet from the end.

NOTE.—All bodies gravitate in an inverse proportion to the distance of the center of gravity.

As 1 is to 6, the center, so is 2 to the answer required.

Man.	Feet.	Men.
As 1 :	6 : :	2

$$\begin{array}{r} 6 \\ 1 \\ \hline 2)6 \\ \hline 3 \end{array}\ \textit{Ans.}$$

A stick of timber, 30 feet long, to be carried by 5 men: two carry at one end, the other three by a lever; how far from the center must the lever be placed that all may carry equally?

Men.	Feet.	Men.
As 2 :	15 : :	3

$$\begin{array}{r} 15 \\ 2 \\ \hline 3)30 \\ \hline 10 \end{array}\ \textit{Ans.}$$

MENSURATION OR PRACTICAL GEOMETRY.

MEASUREMENT OF GRINDSTONES.

Grindstones are sold by the stone, and their contents found as follows:*

RULE.—*To the whole diameter add half of the diameter, and multiply the sum of these by the same half, and this product by the thickness; divide this last number by* 1728, *and the quotient is the contents, or answer required.*

EXAMPLES.

What are the contents of a grindstone 24 inches diameter, and 4 inches thick

$$\frac{\overline{24+12}\times24\times4}{1728}=1 \text{ stone. } Ans.$$

2. What are the contents of a grindstone 36 inches diameter, and 4 inches thick. *Ans.* $2\frac{1}{4}$ stone.

Mensuration of Superficies and Solids.

Superficial measure is that which relates to length and breadth only, not regarding thickness. It is made up of squares, either greater or less, according to the different measures by which the dimensions of the figure are taken or measured. Land is measured by this measure, its dimensions being

*24 inches in diameter, and 4 inches thick make a stone.

usually taken in acres, rods, and links. The contents of boards, also, are found by this measure, their dimensions being taken in feet and inches. Because 12 inches in length make 1 foot of long measure, therefore $12\times12=144$, the square inches in a superficial foot, etc.

NOTE.—Superficial means lying on the surface.

To find the area of a square having equal sides.

RULE.—*Multiply the side of the square into itself, and the product will be the area, or superficial content of the same name with the denomination taken, whether inches, feet, yards, rods, and links, or acres.*

EXAMPLES.

1. How many square feet of boards are contained in the floor of a room which is 20 feet square?

$20\times20=400$ feet, the answer.

2. Suppose a square lot of land measures 36 rods on each side, how many acres does it contain?

$36\times36=1296$ square rods. And $1296\div160=8$ acres, 16 rods, *Ans.*

As 160 square rods make an acre, therefore we divide 1296 by 160 to reduce rods to acres.

N. B.—The shortest way to work this example is, to cancel 36×36 with the divisor 160. Arrange the example as below; (divide both terms by 4×4:)

$$\frac{36\times36}{160} \text{ same as } \frac{9\times9}{10}=8.1 \text{ acres, or 8ac. 16 rods}$$

To measure a parallelogram or long square.

Rule.—*Multiply the length by the breadth, and the product will be the area, or superficial content, in the same name as that in which the dimension was taken, whether inches, feet, or rods, etc.*

EXAMPLES

1. A certain garden, in form of a long square, is 96 feet long, and 54 feet wide; how many square feet of ground are contained in it?

Ans. $96 \times 54 = 5184$ square feet.

2. A lot of land, in form of a long square, is 120 rods in length, and 60 rods wide; how many acres are in it? $120 \times 60 = 7200$ sq. rods. And $7200 \div 160 = 45$ acres, *Ans.*

Note.—The learner must recollect that feet in length, multipled by feet in breadth, produce *square* feet; and the same of the other denominations of lineal measure.

Note.—Both the length and breadth, if not in units of the same denomination, must be made so before multiplying.

3. How many acres are in a field of oblong form, whose length is 14,5 chains, and breath 9,75 chains? *Ans.* 14ac. 0rood, 22rods.

Note.—The Gunter's chain is 66 feet, or 4 rods, long, and contains 100 links. Therefore if dimensions be given in chains and decimals, point off from the product one more decimal place than are

contained in both factors, and it will be acres and decimals of an acre; if in chains and links, do the same, because links are hundredths of chains, and therefore the same as decimals of them. Or, as 1 chain wide, and 10 chains long, or 10 square chains, or 100000 square links, make an acre, it is the same as if you divide the links in the area by 100000.

4. If a board be 21 feet long and 18 inches broad, how many square feet are contained in it?

18 inches=1,5 foot; and $21 \times 1{,}5 = 31{,}5$ ft., *Ans.*

Or, in measuring boards, you may multiply the length in feet by the breadth in inches, and divide the product by 12; the quotient will give the answer in square feet, etc.

Thus, in the last example, $\frac{21 \times 18}{12} = 31\frac{1}{2}$sq. ft., as before.

5. If a board be 8 inches wide, how much in length will make a foot square?

Rule.—*Divide* 144 *by the width; thus,* 8)144

Ans. 18 *in.*

6. If a piece of land be 5 rods wide, how many rods in length will make an acre?

Rule.—*Divide* 160 *by the width, and the quotient will be the length required; thus,*

5)160

Ans. 32 *rods in length.*

NOTE.—When a board, or any other surface, is wider at one end than the other, but yet is of a true taper, you may take the breadth in the middle, or add the widths of both ends together, and halve the sum for the mean width; then multiply the said mean breadth in either case by the length; the product is the answer or area sought.

7. How many square feet in a board, 10 feet long and 13 inches wide at one end, and 9 inches wide at the other?

$$\frac{13+9}{2}=11 \text{ in., mean width.}$$

$$\frac{\overset{\text{ft.}}{10}\times\overset{\text{in.}}{11}}{12}=9\tfrac{1}{6}\text{ft., } \textit{Ans.}$$

8. How many acres are in a lot of land which is 40 rods long, and 30 rods wide at one end, and 20 rods wide at the other?

$$\frac{30+20}{2}=25 \text{ rods, mean width.}$$

Then,

$$\frac{25\times40}{160}=6\tfrac{1}{4} \text{ acres, } \textit{Ans.}$$

9. If a farm lie 250 rods on the road, and at one end be 75 rods wide, and at the other 55 rods wide, how many acres does it contain?

Ans. 101 acres, 2 roods, 10 rods.

N. B.—Always arrange your example as above and cancel the factors common to both terms before multiplying

CASE 3.—*To measure the surface of a triangle.*

DEFINITION.—A triangle is any three-cornered figure which is bounded by three right lines.*

RULE.—*Multiply the base of the given triangle into half its perpendicular hight, or half the base into the whole perpendicular, and the product will be the area.*

EXAMPLES.

1. Required the area of a triangle whose base or longest side is 32 inches, and the perpendicular hight 14 inches.

$14 \div 2 = 7 =$ half the perpendicular. And $32 \times 7 = 224$sq. in., *Ans.*

2. There is a triangular or three-cornered lot of land whose base or longest side is $51\frac{1}{2}$ rods; the perpendicular, from the corner opposite to the base, measures 44 rods; how many acres does it contain?

$44 \div 2 = 22 =$ half the perpendicular.

And $\dfrac{51{,}5 \times 22}{160} = 7$ acres, 13 rods, *Ans.*

Joists and planks are measured by the following:

RULE.—*Find the area of one side of the joist or plank by one of the preceding rules; then multiply it by the thickness in inches, and the last product will be the superficial content.*

* A triangle may be either right-angled or oblique.

EXAMPLES.

1. What is the area, or superficial content, or board measure, of a joist, 20 feet long, 4 inches wide, and 3 inches thick? $\frac{20\times4}{12}\times3=20$ft., *Ans.*

2. If a plank be 32 feet long, 17 inches wide, and 3 inches thick, what is the board measure of it? *Ans.* 136 feet

NOTE.—There are some numbers, the sum of whose squares makes a perfect square; such are 3 and 4, the sum of whose squares is 25, the square root of which is 5; consequently, when one leg of a right-angled triangle is 3, and the other 4, the hypotenuse must be 5. And if 3, 4, and 5, be multiplied by any other numbers, each by the same, the products will be sides of true right-angled triangles. Multiplying them by 2, gives 6, 8, and 10, by 3, gives 9, 12, and 15; by 4, gives 12, 16, and 20, etc.; all which are sides of right-angled triangles. Hence architects, in setting off the corners of buildings, commonly measure 6 feet on one side, and 8 feet on the other; then, laying a 10-foot pole aeross from those two points, it makes the corner a true right-angle.

N. B.—The solutions of the foregoing problems are all very brief by canceling.

To find the area of any triangle when the three sides only are given.

Rule.—*From half the sum of the three sides subtract each side severally; multiply these three remainders and the said half sum continually together; then the square root of the last product will be the area of the triangle.*

EXAMPLE.

Suppose I have a triangular fish-pond, whose three sides measure 400, 348, and 312yds; what quantity of ground does it cover?

Ans. 10 acres, 3 roods, 8+rods.

Note.—If a stick of timber be hewn three square, and be equal from end to end, you find the area of the base, as in the last question, in inches; multiply that area by the whole length, and divide the product by 144, to obtain the solid content.

If a stick of timber be hewn three square, be 12 feet long, and each side of the base 10 inches, the perpendicular of the base being 8⅔ inches, what is its solidity? *Ans.* 3,6+feet.

PROBLEM 1.

The diameter given, to find the circumference.

Rule.—*As* 7 *are to* 22, *so is the given diameter to the circumference; or, more exactly, as* 113 *are to* 355, *so is the diameter to the circumference, etc*

EXAMPLES.

1. What is the circumference of a wheel, whose diameter is 4 feet?

As 7 : 22 : : 4 : 12,57+ft., the circum., *Ans.*

2. What is the circumference of a circle, whose diameter is 35 rods?

As 7 : 22 : : 35 : 110 rods, *Ans.*

NOTE.—To find the diameter when the circumference is given, reverse the foregoing rule, and say, as 22 are to 7, so is the given circumference to the required diameter; or, as 355 are to 113, so is the circumference to the diameter.

3. What is the diameter of a circle, whose circumference is 110 rods?

As 22 : 7 : : 110 : 35 rods, the diam., *Ans.*

CASE 5.—*To find how many solid feet a round stick of timber, equally thick from end to end, will contain, when hewn square.*

RULE.—*Multiply twice the square of its semi-diameter, in inches, by the length in the feet; then divide the product by* 144, *and the quotient will be the answer.*

N. B.—When multiplication and division are combined, always cancel like factors. When the numbers are properly arranged, a few clips with the pencil; and, perhaps, a *trifling* multiplication will suffice.

EXAMPLES.

1. If the diameter of a round stick of timber be 22 inches, and its length 20 feet, how many solid feet will it contain when hewn square?

Half diameter$=11$, and $\dfrac{11\times11\times2\times20}{144}=33,6+$ft. the solidity when hewn square, the answer.

CASE 6.—*To find how many feet of square edged boards, of a given thickness, can be sawn from a log of a given diameter.*

RULE.—*Find the solid content of the log, when made square, by the last case; then say, as the thickness of the board, including the saw calf, is to the solid feet, so is* 12 *inches to the number of feet of boards.*

EXAMPLES.

1. How many feet of square edged boards, $1\frac{1}{4}$ inch thick, including the saw calf, can be sawn from a log 20 feet long, and 24 inches diameter?

$\dfrac{12\times12\times2\times20}{144}=40$ft. solid content when hewn sq.

As $1\frac{1}{4}$: 40 : : 12 : 384 feet, *Ans.*

2. How many feet of square edged boards, $1\frac{1}{2}$ inch thick, including the saw gap, can be sawn from a log 12 feet long, and 18 inches diameter?

Ans. 108 feet.

NOTE.—A short rule for finding the number of feet of one inch boards that a log will make, is to deduct $\frac{1}{4}$ of its diameter in inches, and $\frac{1}{4}$ of its length in feet; then for each inch of diameter that remains, reckon 1 board of the same width as this reduced diameter, and of the same length as this reduced length of the log: thus a log 12 feet long, and 12 inches through, gives 9 boards, 9 feet long, 9 inches wide, or $60\frac{3}{4}$ feet—a log 16 feet long, and 16 inches through, gives 12 boards, 12 inches wide, 12 feet long, or 144 feet.

In measuring timber, however, you may multiply the breadth in inches by the depth in inches, and that product by the length in feet; divide this last product by 144 and the quotient will be the solid content in feet, etc.

How many solid feet does a piece of square timber, or a block of marble contain, if it be 16 inches broad, 11 inches thick, and 20 feet long?

$16 \times 11 \times 20 = 3520$, and $3520 \div 144 = 24{,}4+$ sol. ft.

CASE 8.—*To find the solidity of a cone or pyramid whether round, square, or triangular.*

DEFINITION.—Solids which decrease gradually from the base till they come to a point, are generally called cones or pyramids, and are of various kinds, according to the figure of their bases; round, square, oblong, triangular, etc.; the point at the top is called the vertex, and a line drawn frcm the

vertex, perpendicular to the base, is called the hight of the pyramid.

RULE.—*Find the area of the base, whether round, square, oblong, or triangular, by some one of the fore going rules, as the case may be; then multiply this area by one-third of the hight, and the product will be the solid content of the pyramid.*

EXAMPLES.

1. What is the content of a true-tapered round stick of timber, 24 feet perpendicular length, 15 inches diameter at one end, and a point at the other?

$$\frac{15\times15\times,7854\times8}{144}=9,8175 \text{ solid feet, } \textit{Ans.}$$

To find the solid content of a frustum af a cone.

What is the solid content of a tapering round stick of timber, whose greatest diameter is 13 inches, the least $6\frac{1}{2}$ inches, and whose length is 24 feet, calculating it by both rules?

RULE. 2 —*Multiply each diameter into itself; multiply one diameter by the other; multiply the sum of these products by the lengths; annex two ciphers to the product, and divide it by 382; the quotient will be the content, which divide by 144 for feet as in other cases.*

$$\frac{(13\times13)+(6,5\times6,5)+(13\times6,5)\times2400}{382}=1858,115+$$

And $1858,115\div144=12,903+$ ft, *Ans.*

To find the content of timber in a tree, multiply the square of $\frac{1}{5}$ of the circumference at the middle of the tree, in inches, by twice the length in feet, and the product divided by 144 will be the content, extremely near the truth. In oak, an allowance of $\frac{1}{10}$ or $\frac{1}{12}$ must be made for the bark, if on the tree; in other wood, less trees of irregular growth, must be measured in parts.

To find the solid content of a frustum or segment of a globe.

Definition.—The frustum of a globe is any part cut off by a plane.

Rule.—*To three times the square of the semi-diameter of the base, add the square of the hight; multiply this sum by the hight, and the product again by* .5236; *the last product will be the solid content.*

EXAMPLE.

If the hight of a coal-pit, at the chimney, be 9 feet, and the diameter at the bottom be 24 feet, how many cords of wood does it contain, allowing nothing for the chimney?

$24 \div 2 = 12 =$ h'fdiam. $12 \times 12 \times 3 = 432$. $9 \times 9 = 81$

$$\text{And } \frac{\overline{432+81} \times 9 \times ,5236}{128 = \text{solid feet in a cord.}} \quad 18,886+ \text{cords, } \textit{Ans.}$$

NOTE.—A pile of wood that is 8 feet long, 4 feet high, and 4 feet wide, contains 128 cubic feet, or a cord, and every cord contains 8 *cord-feet;* and as 8 is $\frac{1}{16}$ of 128, every *cord-foot* contains 16 cubic feet; therefore, dividing the cubic feet in a pile of wood by 16, the quotient is the cord-feet; and if cord-feet be divided by 8, the quotient is cords.

NOTE.—If we wish to find the circumference of a tree, which will hew any given number of inches square, we divide the given side of the square by .225, and the quotient is the circumference required.

What must be the circumference of a tree that will make a beam 10 inches square?

NOTE.—When wood is "corded" in a pile 4 feet wide, by multiplying its length by its hight, and dividing the product by 4, the quotient is the cord-feet; and if a load of wood be 8 feet long, and its hight be multiplied by its width, and the product divided by 2, the quotient is the cord-feet.

How many cords of wood in a pile 4 feet wide, 70 feet 6 inches long, and 5 feet 3 inches high?

NOTE.—Small fractions rejected.

To find how large a cube may be cut from any given sphere, or be inscribed in it.

RULE.—*Square the diameter of the sphere, divide that product by* 3, *and extract the square root of the quotient for the answer.*

To find the contents of a round vessel, wider at one end than the other.

RULE.—*Multiply the great diameter by the less; to this product add* $\frac{1}{3}$ *of the square of their difference, then multiply by the hight, and divide as in the last rule.*

Having the diameter of a circle given, to find the area.

RULE.—*Multiply half the diameter by half the circumference, and the product is the area; or, which is the same thing, multiply the square of the diameter by* .7854, *and the product is the area.*

To find the solidity of a sphere or globe.

RULE.—*Multiply the cube of the diameter by* .5236.

To find the convex surface of a sphere or globe.

RULE.—*Multiply its diameter by its circumference.*

To find the solidity of a prism.

RULE—*Multiply the area of the base, or end, by the hight.*

How many wine gallons will a cubical box contain, that is 10 long, 5 feet wide, and 4 feet high?

RULE.—*Take the dimensions in inches; then multiply the length, breadth, and hight together; divide the product by* 282 *for ale gallons,* 231 *for wine gallons, and* 2150 *for bushels.*

I have a piece of timber, 30 inches in diameter; how large a square stick can be hewn from it?

RULE.—*Multiply the diameter by* .7071, *and the product is the side of a square inscribed.*

I have a circular field, 360 rods in circumference; what must be the side of a square field that shall contain the same quantity?

RULE.—*Multiply the circumference by* .282, *and the product is the side of an equal square.*

I have a round field, 50 rods in diameter; what is the side of a square field that shall contain the same area? *Ans.* 44.31135+rods.

RULE.—*Multiply the diameter by* .886, *and the product is the side of an equal square.*

There is a certain piece of round timber, 30 inches in diameter; required the side of an equilateral triangular beam that may be hewn from it.

RULE.—*Multiply the diameter by* .866, *and the product is the side of an inscribed equilateral triangle.*

To find the area of a globe or sphere.

DEFINITION.—A sphere or globe is a round solid body, in the middle or center of which is an imaginary point, from which every part of the surface is equally distant. An apple, or a ball used by children in some of their pastimes, may be called a sphere or globe.

RULE.—*Multiply the circumference by the diameter, and the product will be the area or surface.*

To find the force of the wedge.

RULE.—*As half the breadth or thickness of the head of the wedge is to one of its slanting sides, so is the power which acts against its head to the force produced at its side.*

Suppose 100 pounds to be applied to the head of a wedge that is 2 inches broad, and whose slant is 20 inches long, what force would be affected on each side?

To find the solidity of a cone or pyramid.

RULE.—*Multiply the area of the base by one-third of its height, or vice versa.*

II. TIMBER MEASURE

PROBLEM 1.

To find the superficial contents of a board or plank.

RULE.—*Multiply the length by the breadth.*

NOTE.—When the board is broader at one end than the other, add the breadth of the two ends together, and take half the sum for a mean breadth.

N. B.—When the breadth of the board is in inches, or feet and inches:

RULE.—*Multiply the length of the board, taken in feet, by its breadth taken in inches, and divide this product by 12; the quotient is the contents in square feet.*

PROBLEM III.

To find the solid contents of squared or four-sided Timber.

By the Carpenters' Rule.

As 12 on D : length on C : Quarter girt on D : solidity on C.

Rule I.—*Multiply the breadth in the middle by the depth in the middle, and that product by the length for the solidity.*

Note.—If the tree taper regularly from one end to the other, half the sum of the breadths of the two ends will be the breadth in the middle, and half the sum of the depths of the two ends will be the depth in the middle.

Rule II.—*Multiply the sum of the breadths of the two ends by the sum of the depths, to which add the product of the breadth and depth of each end; one-sixth of this sum multiplied by the length, will give the correct solidity of any piece of squared timber tapering regularly.*

PROBLEM IV.

To find how much in length will make a solid foot, or any other asssigned quantity, of squared timber, of equal dimensions from end to end.

Rule.—*Divide* 1728, *the solid inches in a foot or the solidity to be cut off, by the area of the end in inches, and the quotient will be the length in inches.*

NOTE.—To answer the purpose of the above rule, some carpenters' rules have a little table upon them, in the following form, called a *table of timber measure.*

0	0	0	0	9	0	11	3	9	inches.
144	36	16	9	5	4	2	2	1	feet.
1	2	3	4	5	6	7	8	9	side of the square.

This table shows, that if the side of the square be 1 inch, the length must be 144 feet; if 2 inches be the side of the square, the length must be 36 feet, to make a solid foot.

PROBLEM V.

To find the solidity of round or unsquared timber

RULE I.—*Gird the timber round the middle with a string; one-fourth part of this girt squared and multiplied by the length will give the solidity.*

NOTE.—If the circumference be taken in inches, and the length in feet, divide the last product by 144.

RULE II—By the Table.—*Multiply the area corresponding to the quarter-girt in inches, by the length of the piece of timber in feet, and the product will be the solidity.*

NOTE.—If the quarter girt exceed the table, take half of it, and four times the content thus formed will be the answer.

A TABLE FOR MEASURING TIMBER.

Quarter Girt.	Area.	Quarter Girt.	Area.	Quarter Girt.	Area.
Inches.	Feet.	Inches.	Feet.	Inches.	Feet.
6	.250	12	1.000	18	2.250
6¼	.272	12¼	1.042	18½	2.376
6½	.294	12½	1.085	19	2.506
6¾	.317	12¾	1.129	19½	2.640
7	.340	13	1.174	20	2.777
7¼	.364	13¼	1.219	20½	2.917
7½	.390	13½	1.265	21	3.062
7¾	.417	13¾	1.313	21½	3.209
8	.444	14	1.361	22	3.362
8¼	.472	14¼	1.410	22½	3.516
8½	.501	14½	1.460	23	3.673
8¾	.531	14¾	1.511	23½	3.835
9	.562	15	1.562	24	4.000
9¼	.594	15¼	1.615	24½	4.168
9½	.626	15½	1.668	25	4.340
9¾	.659	15¾	1.722	25½	4.516
10	.694	16	1.777	26	4.694
10¼	.730	16¼	1.833	26½	4.876
10½	.766	16½	1.890	27	5.062
10¾	.803	16¾	1.948	27½	5.252
11	.840	17	2.006	28	5.444
11¼	.878	17¼	2.066	28½	5.640
11½	.918	17½	2.126	29	5.840
11¾	.959	17¾	2.187	29½	6.044
				30	6.250

RULE III—By the Carpenters Rule.—*Measure the circumference of the piece of timber in the middle and take a quarter of it in inches, call this the girt.*

Then set 12 on D, to the length in feet on C, and against the girt in inches on D, you will find the content in feet on C.

EXAMPLE 1.

If a piece of round timber be 18 feet long, and the quarter girt 24 inches, how many feet of timber are contained therein?

24 quarter girt.	
24	
96	
48	*By the Table.*
576 square.	Against 24 stands 4.00
18	Length, 18
4608	Product, 72.00
576	
	Ans. 72 feet.
144)10368(72 feet	
1008	
288	
288	

By the Carpenters' Rule.

12 *on* D: 18 *on* C: 24 *on* D: 72 *on* C.

III. CARPENTERS' AND JOINERS' WORK.

The Carpenters' and Joiners' works, which are measurable, are flooring, partitioning, roofing wainscoting, etc.

1. *Of Flooring.*

Joists are measured by multiplying their breadth by their depth, and that product by their length. They receive various names, according to the position in which they are laid to form a floor, such as trimming joists, common joists, girders, binding joists, bridging joists and ceiling joists.

Girders and joists of floors, designed to bear great weights, should be let into the walls at each end about two-thirds of the wall's thickness.

In boarded flooring, the dimensions must be taken to the extreme parts, and the number of squares of 100 feet must be calculated from these dimensions. Deductions must be made for staircases, chimneys, etc.

Example 1. If a floor be 57 feet 3 inches long, and 28 feet 6 inches broad, how many squares of flooring are there in that room?

By Decimals.	*By Duodecimals.*
57.25	F. I.
28.5	57 : 3
	28 : 6
28625	
45800	456
11450	114
	28 : 7 : 6
100)1631.625 feet.	7 : 0 : 0
Squares 16.31625	16:31 : 7 : 6

Ans. 16 squares and 31 feet.

IV. OF BRICKLAYERS' WORK.

The principal is tiling, slating, walling and chimney work.

1. *Of Tiling or Slating.*

Tiling and slating are measured by the square of 100 feet, as flooring, partitioning and roofing were in the Carpenters' work; so that there is not much difference between the roofing and tiling; yet the tiling will be the most; for the bricklayers sometimes will require to have double measure for hips and valleys.

When gutters are allowed double measure, the way is to measure the length along the ridge-tile, and add it to the content of the roof: this makes an allowance of one foot in breadth, the whole length of the hips or valleys. It is usual also to allow double measure at the eaves, so much as the projector is over the plate, which is commonly about 18 or 20 inches.

Sky-lights and chimney shafts are generally deducted, if they be large, otherwise not.

Example 1. There is a roof covered with tiles, whose depth on both sides (with the usual allowance at the eaves) is 37 feet 3 inches, and the length 45 feet; how many squares of tiling are contained therein?

BY DUODECIMALS.		BY DECIMALS.
FEET.	INCHES.	
37	3	37.25
45	0	45
——		——
185		18625
148		14900
11	3	——
——		16 76.25
16 76	3	

2. *Of Walling.*

Bricklayers commonly measure their work by the rod of $16\frac{1}{2}$ feet, or $272\frac{1}{4}$ square feet. In some places it is a custom to allow 18 feet to the rod; that is, 324 square feet. Sometimes the work is measured by the rod of 21 feet long and 3 feet high, that is, 63 square feet; and then no regard is paid to the thickness of the wall in measuring, but the price is regulated according to the thickness.

When you measure a piece of brick-work, the first thing is to inquire by which of these ways it must be measured; then, having multiplied the length and breadth in feet together, divide the product by the proper divisor, viz.: 272.25, 324 or 63, according to the measure of the rod, and the quotient will be the answer in square rods of that measure.

But, commonly, brick walls that are measured by the rod are to be reduced to a standard thick-

ness of a brick and a-half, which may be done by the following

RULE.—*Multiply the number of superficial feet that are contained in the wall by the number of half bricks which that wall is in thickness; one-third part of that product will be the content in feet.*

The dimensions of a building are generally taken by measuring half round the outside and half round the inside, for the whole length of the wall; this length, being multiplied by the hight, gives the superficies. And to reduce it to the standard thickness, etc., proceed as above. All the vacuities, such as doors, windows, window backs, etc., must be deducted.

To measure any arched way, arched window or door, etc., take the hight of the window or door from the crown or middle of the arch to the bottom or sill, and likewise from the bottom or sill to the spring of the arch; that is, where the arch begins to turn. Then to the latter hight add twice the former, and multiply the sum by the width of the window, door, etc., and one-third of the product will be the area, sufficiently near for practice.

Example 1. If a wall be 72 feet 6 inches long, and 19 feet 3 inches high, and $5\frac{1}{2}$ bricks thick, how many rods of brick work are contained therein, when reduced to the standard?

VII. GLAZIERS' WORK.

Glaziers take their dimensions in feet, inches and eights or tenths, or else in feet and hundredth parts of a foot, and estimate their work by the square foot.

Windows are sometimes measured by taking the dimensions of one pane, and multiplying its superficies by the number of panes. But, more generally, they measure the length and breadth of the window over all the panes and their frames for the length and breadth of the glazing.

Circular or oval windows, as fan lights, etc., are measured as if they were square, taking for their dimensions the greatest length and breadth, as a compensation for the waste of glass and labor in cutting it to the necessary forms.

Example 1. If a pane of glass be 4 feet $8\frac{3}{4}$ inches long, and 1 foot $4\frac{1}{4}$ inches broad, how many feet of glass are in that pane?

BY DUODECIMALS.

FT.	IN.	P.		
4	8	9		
1	4	3		
4	8	9		
1	6	11	0	
	1	2	2	3
6	4	10	2	3

BY DECIMALS.

```
     4.729
     1.354
     -----
     18916
    23645
   14187
  4729
  --------
  6.403066
```

Ans. 6 feet 4 inches.

VIII. PLUMBERS WORK.

Plumbers' work is generally rated at so much per pound, or by the hundred weight of 112 pounds, and the price is regulated according to the value of lead at the time when the work is performed.

Sheet lead, used in roofing, guttering, etc., weighs from 6 to 12 pounds per square foot, according to the thickness, and leaden pipe varies in weight per yard, according to the diameter of its bore in inches.

The following table shows the weight of a square foot of sheet lead, according to its thickness, reckoned in parts of an inch, and the common weight of a yard of leaden pipe corresponding to the diameter of its bore in inches:

Thickness of Lead.	Pounds to a Square foot.	Bore of Leaden Pipe.	Pounds per yard.
$\frac{1}{10}$	5.899	$\frac{3}{4}$	10
$\frac{1}{9}$	6.554	1	12
$\frac{1}{8}$	7.373	$1\frac{1}{4}$	16
$\frac{1}{7}$	8.427	$1\frac{1}{2}$	18
$\frac{1}{6}$	9.831	$1\frac{3}{4}$	21
$\frac{1}{5}$	11.797	2	24

Example 1. A piece of sheet lead measures 16 feet 9 inches in length, and 6 feet 6 inches in breadth; what is its weight at $8\frac{1}{4}$ pounds to a square foot?

BY DUODECIMALS			BY DECIMALS
FEET.	INCHES.		FEET.
16	9		16.75
6	6		6.5
100	6		8375
8	4	6	10050
108	10	6	108.875 *feet.*

Then 1 foot : $8\frac{1}{4}$ pounds : : 108.875 feet : 898.21875 pounds=8 cwt. $2\frac{1}{4}$ pounds nearly.

IX. MASON'S WORK

Masons measure their work sometimes by the foot solid, sometimes by the foot superficial, and sometimes by the foot in length. In taking dimensions they girt all their moldings as joiners do.

The solids consist of blocks of marble, stone pillars, columns, etc. The superficies are pavements, slabs, chimney-pieces, etc.

V. PLASTERERS' WORK.

Plasterers' work is principally of two kinds; namely, plastering upon laths, called *ceiling*, and plastering upon walls or partitions made of framed timber, called *rendering*.

In plastering upon walls, no deductions are made except for doors and windows, because cornices, festoons, enriched moldings, etc., are put on after the room is plastered.

In plastering timber partitions, in large warehouses, etc., where several of the braces and larger timbers project from the plastering, a fifth part is commonly deducted. Plastering between their timbers is generally called rendering between quarters.

Whitening and coloring are measured in the same manner as plastering; and in timbered partitions, one-fourth, or one-fifth of the whole area is commonly added, for the trouble of coloring the sides of the quarters and braces.

Plasterers' work is measured by the yard square, consisting of nine square feet. In arches, the girt round them, multiplied by the length, will give the superficies.

Example 1.—If a ceiling be 59 feet 6 inches long, and 24 feet 6 inches broad; how many yards does that ceiling contain?

PROBLEM I.

To find the solid content of a Dome, having the hight and the dimensions of its base given.

RULE.—*Multiply the area of the base by the hight, and $\frac{2}{3}$ of the product will be the solidity.*

Example 1.—What is the solidity of a dome, in the form of a hemisphere, the diameter of the circular base being 60 feet?

$60^2 \times .7854 = 2827.44$ area of the base.

Then $\frac{2}{3}$ $(2827.44 \times 30) = 56548.8$ cubic feet, *Ans.*

PROBLEM II.

To find the superficies of a dome, having the hight and dimensions of its base given.

RULE.—*Multiply the area of the base by* 2, *and the product will be the superficial content required; or, multiply the square of the diameter of the base by* 1.5708.

FOR AN ELLIPTICAL DOME.—*Multiply the two diameters of the base together, and that product by* 1.5708, *the last product will be the area, sufficiently correct for practical purposes.*

XI. CISTERNS.

Cisterns are large reservoirs constructed to hold water, and to be permanent, should be made either of brick or masonry.

It frequently occurs that they are to be so constructed as to hold given quantities of water, and it then becomes a useful and practical problem to calculate their exact dimensions.

How do you find the number of hogsheads which a cistern of given dimensions will contain?

1st. Find the solid content of the cistern in cubic inches.

2d. Divide the content so found by 14553, and the quotient will be the number of hogsheads.

If the hight of a cistern be given, how do you find the diameter, so that the cistern shall contain a given number of hogsheads?

1st. Reduce the hight of the cistern to inches, and the content to cubic inches.

2d. Multiply the hight by the decimal .7854.

2. Divide the content by the last result, and extract the square root of the quotient, which will be the diameter of the cistern in inches.

EXAMPLE.

If the diameter of a cistern be given, how do you find the hight, so that the cistern shall contain a given number of hogsheads?

1st. Reduce the content to cubic inches.

2d. Reduce the diameter to inches, and then multiply its square by the decimal .7854.

3d. Divide the content by the last result, and the quotient will be the hight in inches.

XII. BINS FOR GRAIN.

Having any number of bushels, how then will you find the corresponding number of cubic feet?

Increase the number of bushels one-fourth itself, and the result will be the number of cubic feet.

How will you find the number of bushels which a bin of a given size will hold?

Find the content of the bin in cubic feet; then diminish the content by one-fifth, and the result will be the content in bushels.

How will you find the dimensions of a bin which shall contain a given number of bushels?

Increase the number of bushels one-fourth itself, and the result will show the number of cubic feet which the bin will contain. Then, when two dimensions of the bin are known, divide the last result by their product, and the quotient will be the other dimension.

A Log Table.—Showing the number of feet of boards any log will make whose diameter is from 15 to 36 inches at the smallest end, and from 10 to 15 feet in length.

Diameter in Inches.	10 Feet in length.	Diameter in Inches.	11 Feet in length.	Diameter in Inches.	12 Feet in length.	Diameter in Inches.	13 Feet in length.	Diameter in Inches.	14 Feet in length.	Diameter in Inches.	15 Feet in length.
15	90	15	99	15	108	15	117	15	126	15	135
16	100	16	110	16	120	16	130	16	140	16	150
17	125	17	137	17	150	17	160	17	175	17	187
18	155	18	170	18	186	18	201	18	216	18	232
19	165	19	176	19	198	19	214	19	230	19	247
20	172	20	189	20	206	20	263	20	246	20	258
21	184	21	202	21	220	21	238	21	256	21	276
22	194	22	212	22	232	22	263	22	294	22	291
23	219	23	240	23	278	23	315	23	332	23	333
24	250	24	276	24	300	24	325	24	350	24	375
25	280	25	308	25	336	25	364	25	392	25	420
26	299	26	323	26	346	26	375	26	404	26	448
27	327	27	367	27	392	27	425	27	457	27	490
28	360	28	396	28	432	28	462	28	504	28	540
29	376	29	414	29	451	29	488	29	526	29	564
30	412	30	452	30	494	30	535	30	576	30	618
31	428	31	471	31	513	31	558	31	602	31	642
32	451	32	496	32	541	32	587	32	631	32	676
33	490	33	539	33	588	33	637	33	686	33	735
34	532	34	585	34	638	34	691	34	744	34	798
35	582	35	640	35	698	35	752	35	805	45	863
36	593	36	657	36	717	36	821	36	836	36	889

TABLE FOR BANKING AND EQUATION.

Showing the number of days from any date in one month to the same date in any other month.

Example. How many days from the 2d of February to the 2d of August? Look for February at the left hand, and August at the top—in the angle is 181. In leap year, add one day if February be included.

From / To	Jan.	Feb	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Jan..........	365	31	59	90	120	151	181	212	243	273	304	334
Feb..........	334	365	28	59	89	120	150	181	212	242	273	303
March......	306	337	365	31	61	92	122	153	184	214	245	275
April........	275	306	334	365	30	61	91	122	153	183	214	244
May.........	245	276	304	335	365	31	61	92	123	153	184	214
June.........	214	245	273	304	334	365	30	61	92	122	153	183
July.........	184	215	243	274	304	335	365	31	62	92	123	153
Aug	153	184	212	243	273	304	334	365	31	61	92	122
Sept	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov..........	61	92	120	151	181	212	242	273	304	334	365	30
Dec	31	62	90	121	151	182	212	243	274	304	335	365

TABLE SHOWING DIFFERENCE OF TIME AT 12 O'CLOCK (NOON) AT NEW YORK.

New York.........	12.00 N.	Boston............	12.12 P. M.
Buffalo............	11.40 A. M.	Quebec............	12.12 "
Cincinnati	11.18 "	Portland.........	12.15 "
Chicago............	11.07 "	London	4.55 "
St. Louis..........	10.55 "	Paris	5.05 "
San Francisco...	8.45 "	Rome................	5.45 "
New Orleans.....	10.56 "	Constantinople	6.41 "
Washington......	11.48 "	Vienna	6 00 "
Charleston	11.36 "	St. Petersburg..	6.57 "
Havana............	11.25 "	Pekin, night...	12.40 A. M.

TROY WEIGHT.

By this weight gold, silver, platina and precious stones, except diamonds, are estimated.

20 Mites...... 1 Grain.	20 Pennywts...... 1 Ounce.
20 Grains.... 1 Pennywt.	12 Ounces.......... 1 Pound.

Any quantity of gold is supposed to be divided

into 24 parts, called *carats.* If pure, it is said to be 24 carats fine; if there be 22 parts of pure gold and 2 parts of alloy, it is said to be 22 carats fine The standard of American coin is nine-tenths pure gold, and is worth $20.67. What is called the *new standard*, used for watch cases, etc., is 18 carats fine. The term carat is also applied to a weight of $3\frac{1}{2}$ grains troy, used in weighing diamonds; it is divided into 4 parts, called *grains;* 4 grains troy are thus equal to 5 grains diamond weight.

APOTHECARIES' WEIGHT—USED IN MEDICAL PRESCRIPTIONS.

The pound and ounce of this weight are the same as the pound and ounce troy, but differently divided.

20 Grains Troy...1 Scruple.	8 Drachms...1 Ounce Troy.
8 Scruples........1 Drachm.	12 Ounces....1 Pound Troy.

Druggists *buy* their goods by avoirdupois weight.

AVOIRDUPOIS WEIGHT.

By this weight all goods are sold except those named under troy weight.

$27\frac{11}{32}$ Grains....................................	1 Dram.
16 Drams....................................	1 Ounce.
16 Ounces....................................	1 Pound.
28 Pounds....................................	1 Quarter.
4 Quarters or 100 pounds............	1 Hundred Weight.
20 Hundred Weight......................	1 Ton.

The grain avoirdupois, though never used, is the same as the grain in troy weight. 7,000 grains make the avoirdupois pound, and 5,760 grains the

troy pound. Therefore, the troy pound is less than the avoirdupois pound in the proportion of 14 to 17, nearly; but the troy ounce is greater than the avoirdupois ounce in the proportion of 79 to 72, nearly. In times past it was the custom to allow 112 pounds for a hundred weight, but usage, as well as the laws of a majority of the States, at the present time call 100 pounds a hundred weight.

APOTHECARIES' FLUID MEASURE.

60 Minims	1 Fluid Drachm.
8 Fluid Drachms	1 Ounce (Troy).
16 Ounces (Troy)	1 Pint.
8 Pints	1 Gallon.

MEASURE OF CAPACITY FOR ALL LIQUIDS.

5 Ounces Avoirdupois of water make 1 Gill.

4 Gills	1 Pint	$= 34\frac{2}{3}$ Cubic Inches (nearly).
2 Pints	1 Quart	$= 69\frac{1}{3}$ do
4 Quarts	1 Gallon	$=277\frac{1}{4}$ do

$31\frac{1}{2}$ Gallons	1 Barrel,
42 Gallons	1 Tierce.
63 Gallons, or 2 bbls	1 Hogshead.
2 Hogsheads	1 Pipe or Butt.
2 Pipes	1 Tun.

The gallon must contain exactly 10 pounds avoirdupois, of pure water, at a temperature of 62°, the barometer being at 30 inches. It is the standard unit of measure of capacity for liquids and dry goods of every description, and is $\frac{1}{5}$ larger than the old wine measure, $\frac{1}{32}$ larger than the old

dry measure, and $\frac{1}{60}$ less than the old ale measure. The wine gallon must contain 231 cubic inches.

MEASURE OF CAPACITY FOR ALL DRY GOODS.

4 Gills..............	1 pint	=	$34\frac{2}{3}$ cubic inchs(nearly)
2 Pints..............	1 quart	=	$69\frac{1}{3}$ cubic inches.
4 Quarts...........	1 gallon	=	$277\frac{1}{4}$ cubic inches.
2 Gallons	1 peck	=	$554\frac{1}{2}$ cubic inches.
4 Pecks, or 8 gals.	1 bushel	=	$2150\frac{1}{2}$ cubic inches.
8 Bushels..........	1 quarter	=	$10\frac{1}{4}$ cubic feet (nearly).

When selling the following articles a barrel weighs as here stated:

For rice, 600 lbs.; flour, 196 lbs.; powder, 25 lbs.; corn, as bought and sold in Kentucky, Tennessee, etc., 5 bushels of shelled corn—as bought and sold at New Orleans, a flour-barrel full of ears; potatoes, as sold in New York, a barrel contains $2\frac{1}{4}$ bushels; pork, a barrel is 200 lbs., distinguished in quality by "clear," "mess," "prime;" a barrel of beef is the same weight.

The legal bushel of America is the old Winchester measure of 2,150.42 cubic inches. The imperial bushel of England is 2,218.142 cubic inches, so that 32 English bushels are about equal to 33 of ours.

Although we are all the time talking about the price of grain, etc., by the bushel, we sell by weight, as follows:

Wheat, beans, potatoes, and clover-seed, 60 lbs

to the bushel; corn, rye, flax-seed, and onions, 56 lbs.; corn on the cob, 70 lbs.; buckwheat, 52 lbs.; barley 48 lbs.; hemp-seed, 44 lbs.; timothy-seed, 45 lbs.; castor beans, 46 lbs.; oats, 35 lbs.; bran, 20 lbs.; blue-grass seed, 14 lbs.; salt—the real weight of coarse salt is 85 lbs.; dried apples, 24 lbs.; dried peaches, 33 lbs., according to some rules, but others are 22 lbs. for a bushel, while in Indiana, dried apples and peaches are sold by the heaping bushel; so are potatoes, turnips, onions, apples, etc., and in some sections oats are heaped. A bushel of corn in the ear is three heaped half bushels, or four even full.

In Tennessee a hundred ears of corn is sometimes counted as a bushel.

A hoop $18\frac{1}{2}$ inches diameter, 8 inches deep, holds a Winchester bushel. A box, 12 inches square, 7 and $7\frac{1}{32}$ deep, will hold half a bushel A heaping bushel is 2,815 cubic inches.

CLOTH MEASURE.

$2\frac{1}{4}$ Inches..	1 nail.
4 Nails........................	1 quarter of a yard.
4 Quarters	1 yard.

FOREIGN CLOTH MEASURE.

$2\frac{1}{2}$ Quarters.................	1 Ell Hamburgh.
3 Quarters..................	1 Ell Flemish.
5 Quarters..................	1 Ell English
6 Quarters..................	1 Ell French.

MEASURE OF LENGTH.

12 Inches....................	1 foot.
3 Feet	1 yard.
$5\frac{1}{2}$ Yards..................	1 rod, pole, or perch.
40 Poles	1 furlong.
8 Furlongs, or 1,760 yds,	1 mile.
$69\frac{1}{15}$ Miles...............	1 degree of a great circle of the earth.

By scientific persons and revenue officers, the inch is divided into *tenths, hundredths, etc.* Among mechanics, the inch is divided into *eighths.* The division of the inch into 12 parts, called lines, is not now in use.

A standard English mile, which is the measure that we use, is 5,280 feet in length, 1,760 yards, or 320 rods. A strip, one rod wide and one mile long, is two acres. By this it is easy to calculate the quantity of land taken up by roads, and also how much is wasted by fences.

GUNTER'S CHAIN.

USED FOR LAND MEASURE

$7\frac{92}{100}$ Inches..............................	1 Link.
100 Links, or 66 feet, or 4 poles........	1 Chain.
10 Chains long by 1 broad, or 10 square chains..................	1 Acre.
80 Chains................................	1 Mile.

SURFACE MEASURE.

144 Sq. inches	1 sq. foot	40 Sq. perches	1 rood
9 Sq. feet	1 sq. yard	4 Roods.......	1 acre
$80\frac{1}{4}$ Sq. yards	1 sq. rd or prch	640 Acres	1 sq. mile

Measure 209 feet on each side, and you have a square acre, within an inch.

The following gives the comparative size, in square yards, of acres in different countries:

English acre, 4,840 square yards; Scotch, 6,150; Irish, 7,840; Hamburgh, 11,545; Amsterdam, 9,722; Dantzic, 6,650; France (hectare), 11,960; Prussia (morgen), 3,053.

This difference should be borne in mind in reading of the products per acre in different countries. Our land measure is that of England.

GOVERNMENT LAND MEASURE.

A Township—36 sections, each a mile square.

A section—640 acres.

A quarter section, half a mile square—160 acres.

An eighth section, half a mile long, north and south, and a quarter of a mile wide—80 acres.

A sixteenth section, a quarter of a mile square—40 acres.

The sections are all numbered 1 to 36, commencing at the north-east corner, thus:

6	5	4	3	2	NW | NE SW | SE
7	8	9	10	11	12
18	17	16*	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

The sections are all divided in quarters, which are named by the cardinal points, as in section 1. The quarters are divided in the same way. The description of a forty-acre lot would read: The south half of the west half of the south-west quarter of section 1 in township 24, north of range 7 west, or as the case might be; and sometimes will fall short, and sometimes overrun the number of acres it is supposed to contain.

SQUARE MEASURE—FOR CARPENTERS, MASONS, ETC.

144 Sq Inches...............	1 Sq Foot.
9 Sq Ft, or 1,296 Sq In.	1 Sq Yard.
100 Sq Feet.....................	1 Sq of Flooring, Roofing, etc.
30$\frac{1}{4}$ Sq Yards...............	1 Sq Rod.
36 Sq Yards....	1 Rood of Building.

* School section.

GEOGRAPHICAL OR NAUTICAL MEASURE.

6 Feet..................	1 Fathom.
110 Fathoms or 660 ft.	1 Furlong.
$6075\frac{4}{5}$ Feet...............	1 Nautical Mile.
3 Nautical Miles......	1 League.
20 Leagues or 60 Geo. Miles...	1 Degree.
360 Degrees..........	The earth's circumference $=24{,}855\frac{1}{2}$ miles, nearly.

The nautical mile is $795\frac{4}{5}$ feet longer than the common mile.

MEASURE OF SOLIDITY.

1728 Cubic Inches............	1 Cubic Foot.
27 Cubic Feet................	1 Cubic Yard.
16 Cubic Feet................	1 Cord Foot, or a ft of wood.
8 Cord ft or 128 Cubic ft..	1 Cord.
40 ft of round or 50 ft of hewn timber...	1 Ton.
42 Cubic Feet..............	1 Ton of Shipping.

ANGULAR MEASURE, OR DIVISIONS OF THE CIRCLE.

60 Seconds........	1 Minute.	30 Degrees......	1 Sign.
60 Minutes.......	1 Degree.	90 Degrees......	1 Quadrant.

360 Degrees......... 1 Circumference.

MEASURE OF TIME.

60 Seconds..........................	1 Minute
60 Minutes..........................	1 Hour.
24 Hours............................	1 Day.
7 Days..............................	1 Week.
28 Days.............................	1 Lunar Month.
28, 29, 30 or 31 Days............	1 Cal. Month.
12 Cal. Months....................	1 Year.
365 Days............................	1 Com. Year.
366 Days............................	1 Leap Year.
$365\frac{1}{4}$ Days............................	1 Julian Year.
365 D., 5 H., 48 M., 49 S......	1 Solar Year.
365 D., 6 H., 9 M., 12 S......	1 Siderial Year

ROPES AND CABLES.

6 Feet	1 Fathom
120 Feet	1 Cable Length.

Miscellaneous Important Facts about Weights and Measures.

BOARD MEASURE.

Boards are sold by superficial measure, at so much per foot of one inch or less in thickness, adding one-fourth to the price for each quarter-inch thickness over an inch.

GRAIN MEASURE IN BULK.

Multiply the width and length of the pile together, and that product by the hight, and divide by 2,150, and you have the contents in bushels.

If you wish the contents of a pile of ears of corn, or roots, in heaped bushels, ascertain the cubic inches and divide by 2,818.

A TON WEIGHT.

In this country a ton is 2,000 pounds. In most places a ton of hay, etc., is 2,240 pounds, and in some places that foolish fashion still prevails of weighing all bulky articles sold by the tun, by the "long weight," or tare of 12 lbs. per cwt.

A tun of round timber is 40 feet; of square timber, 54 cubic feet.

A quarter of corn or other grain sold by the bushel is eight imperial bushels, or quarter of a tun.

A ton of liquid measure is 252 gallons.

BUTTER

Is sold by avoirdupois weight, which compares with troy weight as 144 to 175; the troy pound being that much the lightest. But 175 troy ounces equal 192 of avoirdupois.

A firkin of butter is 56 lbs.; a tub of butter is 84 lbs.

THE KILOGRAMME OF FRANCE

Is 1000 grammes, and equal to 2 lbs. 2 oz. 4 grs. avoirdupois.

A BALE OF COTTON,

In Egypt, is 90 lbs.; in America a commercial bale is 400 lbs.; though put up to vary from 280 to 720, in different localities.

A bale or bag of Sea Island cotton is 300 lbs.

WOOL.

In England, wool is sold by the sack or boll, of 22 stones, which, at 14 lbs. the stone, is 308 lbs.

A pack of wool is 17 stone, 2 lbs., which is rated as a pack load for a horse. It is 240 lbs. A tod of wool is 2 stones of 14 lbs. A wey of wool is $6\frac{1}{2}$ tods. Two weys, a sack. A clove of wool is half a stone.

THE STONE WEIGHT

So often spoken of in English measures, is 14 lbs., when weighing wool, feathers, hay, etc.; but a stone of beef, fish, butter, cheese, etc., is only 8 pounds.

HAY.

In England, a truss, when new, is 60 lbs., or 56 lbs. of old hay. A truss of straw, 40 lbs. A load of hay is 36 trusses.

In this country, a load is just what it may happen to weigh; and a tun of hay is either 2,000 lbs. or 2,240 lbs., according to the custom of the locality. A bale of hay is generally considered about 300 lbs., but there is no regularity in the weight. A cube of a solid mow, 10 feet square, will weigh a tun.

A LAST

Is an English measure of various articles.

A last of soap, ashes, herrings, and some similar things, is 2 barrels.

A last of corn is 10 quarters.

A last of gunpowder, 24 barrels.

A last of flax or feathers, 1,700 lbs.

A last of wool, 12 sacks.

A SCOTCH PINT

Contains 105 cubic inches, and is equal to four English pints. 21¼ Scotch pints make a farlot of wheat.

COAL.

A chaldron is $58\frac{2}{3}$ cubic feet, or by measure, 36 heaped bushels. A heaped bushel of anthracite coal weighs 80 lbs., making 2,880 lbs. to a chaldron.

WOOD.

A cord of wood is 128 solid feet, in this country and England. In France it is 576 feet. We cord wood 4 feet long, in piles 4 feet by 8.

In New Orleans wood is retailed by the pound, and to a limited extent here. It is also sold by the barrel. A load of wood in New York is $42\frac{2}{3}$ cubic feet, or one-third of a cord.

Wood is sold in England by the stack, skid, quintal, billet, and bundle.

A stack is 108 solid feet, and unusually piled 12 feet long, 3 feet high, and 3 feet wide.

A quintal of wood is 100 lbs.

A skid is a round bundle of sticks, 4 feet long A one-notch skid girts 16 inches. A two-notch skid, 23 inches. A three-notch skid, 28 inches. A four-notch skid, 33 inches. A five-notch skid, 38 inches.

A billet of wood is a bundle of sticks, 3 feet long, and girts 7, 10, or 14 inches, and these bundles sell by the score or hundred. A score is 20, and comes from the count by tally, or marks.

Faggots of wood are bundles of brush 3 feet long, two round. A load of faggots is 50 bundles.

All wood should be sold by the pound.

CAPACITY OF CISTERNS OR WELLS.

Tabular view of the number of gallons contained in the clear between the brickwork for each ten inches of depth:

DIAMETER.	GALLONS.	DIAMETER.	GALLONS.
2 feet equal.........	19	8 feet equal.........	313
2½ "	30	8½ " "	353
3 " '	44	9 ' "	396
3½ " "	60	9½ ' "	461
4 ' "	78	10 " "	489
4½ " "	97	11 " "	592
5 " "	122	12 " "	705
5½ " "	148	13 " "	827
6 " "	176	14 " "	959
6½ " "	207	15 " "	1101
7 " "	240	20 " "	1958
7½ " "	275	25 " "	3059

TO MEASURE CORN IN THE CRIB.

Corn is generally put up in cribs made of rails, but the rule will apply to a crib of any size or kind.

Two cubic feet of good, sound, dry corn in the ear, will make a bushel of shelled corn. To get then, the quantity of shelled corn in a crib of corn in the ear, measure the length, breadth, and night of the crib, *inside the rail;* multiply the length by

the breadth, and the product by the night; then divide the result by two, and you have the number of bushels of shelled corn in the crib.

In measuring the hight, of course, the hight of the corn is intended. And there will be found to be a difference in measuring corn in this mode, between fall and spring, because it shrinks very much in the winter and spring, and settles down.

RULES FOR DETERMINING THE WEIGHT OF LIVE CATTLE.

Measure in inches the girth round the breast, just behind the shoulder-blade, and the length of the back from the tail to the forepart of the shoulder-blade. Multiply the girth by the length, and divide by 144. If the girth is less than three feet, multiply the quotient by 11; if between three feet and five feet, multiply by 16; if between five feet and seven feet, multiply by 23; if between seven and nine feet, multiply by 31. If the animal is lean, deduct $\frac{1}{20}$ from the result.

Take the girth and length in feet, multiply the square of the girth by the length, and multiply the product by 3.36. The result will be the answer in pounds. The live weight, multiplied by .605 gives a near approximation to the net weight.

ASTRONOMICAL CALCULATIONS.

A scientific method of telling immediately what day of the week any date transpired or will transpire, from the commencement of the Christian Era, for the term of three thousand years.

MONTHLY TABLE.

The ratio to add for each month will be found in the following table:

Ratio of June is..............0	Ratio of October is.........3
Ratio of September is......1	Ratio of May is..............4
Ratio of December is.......1	Ratio of August is..........5
Ratio of April is...........2	Ratio of March is...........6
Ratio of July is..............2	Ratio of February is.......6
Ratio of January is.........3	Ratio of November is......6

NOTE.—On Leap Year the ratio of January is 2, and the ratio of February is 5. The ratio of the other ten months do not change on Leap Years.

CENTENNIAL TABLE.

The ratio to add for each century will be found in the following table:

Christian Era.	200,	900,	1800,	2200,	2600,	3000,	ratio is.........0
	300,	1000,	......	......	......	......	ratio is..........6
	400,	1100,	1900,	2300,	2700,	......	ratio is..........5
	500	1200,	1600,	2000,	2400,	2800,	ratio is..........4
	600	1300,	......	......	......	......	ratio is..........3
000,	700,	1400,	1700,	2100,	2500,	2900,	ratio is..........2
100,	800,	1500,	......	......	......	......	ratio is.........1

NOTE.—The figure opposite each century is its ratio; thus the ratio for 200, 900, etc., is 0. To find the ratic of any century, first find the century in the above table, then run the eye along the line until you arrive at the end; the small figure at the end is its ratio.

METHOD OF OPERATION.

RULE.*—*To the given year add its fourth part, rejecting the fractions; to this sum add the day of the month; then add the ratio of the month and the ratio of the century. Divide this sum by 7; the remainder is the day of the week, counting Sunday as the first, Monday as the second, Tuesday as the third, Wednesday as the fourth, Thursday as the fifth, Friday as the sixth, Saturday as the seventh; the remainder for Saturday will be 0 or zero.*

EXAMPLE 1.—Required the day of the week for the 4th of July, 1810.

To the given year, which is..............................	10
Add its fourth part, rejecting fractions...............	2
Now add the day of the month, which is............	4
Now add the ratio of July, which is...	2
Now add the ratio of 1800, which is....	0
Divide the whole sum by 7.	7 | 18—4
	2

We have 4 for a remainder, which signifies the fourth day of the week, or Wednesday.

*When dividing the year by 4, always leave off the centuries. We divide by 4 to find the number of Leap Years.

NOTE.—In finding the day of the week for the present century, no attention need be paid to the *centennial ratio*, as it is 0.

EXAMPLE 2.—Required the day of the week for the 2d of June, 1805.

To the given year, which is	5
Add its fourth part, rejecting fractions	1
Now add the day of the month, which is	2
Now add the ratio of June, which is	0
Divide the whole sum by 7.	7 | 8—1
	1

We have 1 for a remainder, which signifies the first day of the week, or Sunday.

The Declaration of American Independence was signed July 4, 1776. Required the day of the week.

To the given year, which is	76
Add its fourth part, rejecting fractions	19
Now add the day of the month, which is	4
Now add the ratio of July, which is	2
Now add the ratio of 1700, which is	2
Divide the whole sum by 7.	7 | 103—5
	14

We have 5 for a remainder, which signifies the fifth day of the week, or Thursday.

The Pilgrim Fathers landed on Plymouth Rock Dec. 20, 1620. Required the day of the week.

To the given year, which is	20
Add its fourth part, rejecting fractions	5
Now add the day of the month, which is	20
Now add the ratio of December, which is	1
Now add the ratio of 1600, which is	4
Divide the whole sum by 7.	7 | 50—1
	7

We have 1 for a remainder, which signifies the first day of the week, or Sunday.

On what day will happen the 8th of January, 1815? *Ans.* Sunday.

On what day will happen the 4th of May, 1810?

On what day will happen the 3d of December, 1423? *Ans.* Friday.

On what day of the week were you born?

The earth revolves round the sun once in 365 days, 5 hours, 48 minutes, 48 seconds; this period is, therefore, a *Solar* year. In order to keep pace with the solar year, in our reckoning, we make every fourth to contain 366 days, and call it Leap Year. Still greater accuracy requires, however, that the leap day be dispensed with three times in every 400 years. Hence, every year (except the centennial years) that is divisible by 4 is a *Leap Year*, and every centennial year that is divisible by 400 is also a *Leap Year*. The next centennial year that will be a Leap Year is 2000

For the practical convenience of those who have occasion to refer to mensuration, we have arranged the following useful table of multiples. It covers the whole ground of practical geometry, and should be studied carefully by those who wish to be skilled in this beautiful branch of mathematics:

TABLE OF MULTIPLES.

Diameter of a circle × 3.1416 = Circumference.
Radius of a circle × 6.283185 = Circumference.
Square of the radius of a circle × 3.1416 = Area.
Square of the diameter of a circle × 0.7854 = Area.
Square of the circumference of a circle × 0.07958 = Area.
Half the circumference of a circle × by half its diameter = Area.
Circumference of a circle × 0.159155 = Radius.
Square root of the area of a circle × 0.56419 = Radius.
Circumference of a circle × 0.31831 = Diameter.
Square root of the area of a circle × 1.12838 = Diameter.
Diameter of a circle × 0.86 = Side of inscribed equilateral triangle.
Diameter of a circle × 0.7071 = Side of an inscribed square.
Circumference of a circle × 0.225 = Side of an inscribed square.
Circumference of a circle × 0.282 = Side of an equal square.
Diameter of a circle × 0.8862 = Side of an equal square.
Base of a triangle × by ½ the altitude = Area.
Multiplying both diameters and .7854 together = Area of an ellipse.
Surface of a sphere × by ⅙ of its diameter = Solidity.
Circumference of a sphere × by its diameter = Surface.
Square of the diameter of a sphere × 3.1416 = Surface.
Square of the circumference of a sphere × 0.3183 = Surface.
Cube of the diameter of a sphere × 0.5236 = Solidity.
Cube of the radius of a sphere × 4.1888 = Solidity.
Cube of the circumference of a sphere × 0.016887 = Solidity.
Square root of the surface of a sphere × 0.56419 = Diameter.
Square root of the surface of a sphere × 1.772454 = Circumference.
Cube root of the solidity of a sphere × 1.2407 = Diameter.
Cube root of the solidity of a sphere × 3.8978 = Circumference.
Radius of a sphere × 1.1547 = Side of inscribed cube.
Square root of (⅓ of the square of) the diameter of a sphere = Side of inscribed cube.
Area of its base × by ⅓ of its altitude = Solidity of a cone or pyramid, whether round, square, or triangular.
Area of one of its sides × 6 = Surface of a cube.
Altitude of trapezoid × ½ the sum of its parallel sides = Area.

CONTENTS.

The peculiarities which the student will here find in Multiplication, Interest, Mensuration, and in Averaging Accounts are not all new. Indeed there can be nothing new in principle; but, as far as the author's knowledge extends, he is not aware that these Abbreviations have ever been collected in any Arithmetical work. The impression seems to have been, that the people could not comprehend Arithmetical brevity, nor appreciate Mathematical beauty; but the author, thinking otherwise, presents this brief, yet comprehensive work to the public, with the full assurance that who ever will pay due attention to the subject will be highly gratified and abundantly rewarded.